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Review

Unsteady free convective flow past a semi-infinite vertical plate with uniform heat and mass flux

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An exact solution to the problem of free convective flow past a semi-infinite vertical plate with uniform heat and mass flux has been analyzed. The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration fields are studied for different physical parameters. It was observed that the velocity increases with increasing values of thermal Grashof number or solutal Grashof number. It was also observed that the velocity decreases with increasing values of Prandtl number or the Schmidt number.

Keywords: Free convection, heat and mass flux, vertical plate.

INTRODUCTION

The effect of free convection flow of a viscous incompressible fluid past an infinite vertical plate has many important technological applications in the astrophysical, geophysical and engineering problems. Siegel (958) was the first to study the transient free convective flow past a semi-infinite vertical plate by integral method. The same problem was studied by Gebhart (1961) by an approximate method. Soundalgekar (1977) presented convection effects on the Stokes problem for infinite vertical plate

The effect of heat transfer effects on unsteady free convective flow with uniform heat and mass flux is not studied in the literature. Therefore, it is proposed to study of heat transfer effects on unsteady free convective flow past a semi-infinite vertical plate in the presence of uniform heat and mass flux. The dimensionless governing equations are solved using the Laplace transform technique.

Nomenclature

C', species concentration in the fluid; c_p, specific heat at constant pressure; D, mass diffusion coefficient; Gr, thermal Grashof number; Gc, solutal Grashof number; g, acceleration due to gravity; k, thermal conductivity; $\Pr$, Prandtl number; Sc, Schmidt number; T', temperature of the fluid near the plate; t', time; t, dimensionless time; u', velocity of the fluid in the x'-direction; u_0, velocity of the plate; x', y', coordinates along and normal to the plate respectively; y, dimensionless coordinate axis normal to the plate; q', constant heat flux at the plate; j'', mass flux per unit area.

Greek symbols

β, volumetric coefficient of thermal expansion; β^*, volumetric coefficient of expansion with concentration; μ, coefficient of viscosity; ν, kinematic viscosity; ρ, density of the fluid; θ, dimensionless temperature; ϕ, dimensionless concentration; η, similarity parameter; $erfc$, complementary error function

Subscript

∞, free stream condition

Mathematical analysis

An unsteady flow of a viscous incompressible fluid past an impulsively started infinite vertical plate with heat and mass flux has been considered. The x'- axis is taken along the plate in the vertical upward direction and y'-axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with uniform velocity u_0, the plate temperature and concentration level raised at an uniform rate. Then under the usual Boussinesq's approximation the unsteady flow is governed by the following equations:

Conservation of Momentum

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) \qquad (1)$$

Conservation of Energy (Heat)

$$\rho c_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} \qquad (2)$$

Conservation of Species (Concentration)

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \qquad (3)$$

With the following initial and boundary conditions:

$t \le 0\ :\ u = 0,\ \ T' = T'_\infty,\ \ \ C' = C'_\infty$ for all y'

$$t > 0\ :\ u = u_0,\ \ \frac{\partial T'}{\partial t'} = -\frac{q'}{k},\ \ \frac{\partial C'}{\partial t'} = -\frac{j''}{D} \quad at \quad y' = 0 \qquad (4)$$

On introducing the following non-dimensional quantities:

$$u = \frac{u'}{u_0},\ t = \frac{t' u_0^2}{\nu},\ \ y = \frac{y u_0}{\nu},\ Gr = \frac{g\beta q'\nu^2}{k u_0^4},\ Gc = \frac{g\beta^* j''\nu^2}{D u_0^4},$$

$$\theta = \frac{T' - T'_\infty}{q'\nu / k u_0},\ \ \Pr = \frac{\mu c_p}{k},\ \ \phi = \frac{C' - C'_\infty}{j''\nu / D u_0^4},\ Sc = \frac{\nu}{D}$$

(5)

In equations (1) to (4), we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gc\phi \qquad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \qquad (7)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} \qquad (8)$$

The initial and boundary conditions in non-dimensional quantities are:

$u = 0,\ \ \theta = 0,\ \ \phi = 0$ for all $y, t \le 0$

$$t > 0 : u = 1,\ \ \frac{\partial \theta}{\partial y} = -1,\ \ \frac{\partial \phi}{\partial y} = -1 \quad at \quad y = 0 \qquad (9)$$

$$u \to 0,\ \ \theta \to 0,\ \ \phi \to 0 \quad as \quad y \to \infty$$

All the physical variables are defined in the nomenclature.

Method of solution

The dimensionless governing equations (6) to (8) subject to the boundary conditions (9), are solved by the usual Laplace transform technique. The solutions are derived as:

$$\phi\left(y,t\right) = 2\sqrt{t}\left[\frac{\exp(-\eta^2 Sc)}{\sqrt{\pi}\sqrt{Sc}} - \eta erfc(\eta\sqrt{Sc})\right] \qquad (10)$$

$$\theta\left(y,t\right) = 2\sqrt{t}\left[\frac{\exp(-\eta^2 \Pr)}{\sqrt{\pi}\sqrt{\Pr}} - \eta erfc(\eta\sqrt{\Pr})\right] \qquad (11)$$

$$u(y,t) = erfc(\eta) + \frac{Grt\sqrt{t}}{3(\Pr-1)\sqrt{\Pr}}\left[\begin{array}{l}\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2) - \frac{4}{\sqrt{\pi}}(1+\eta^2\Pr)\exp(-\eta^2\Pr) \\ -\eta\left(6+4\eta^2\right)erfc(\eta) + \eta\sqrt{\Pr}\left(6+4\eta^2\Pr\right)erfc(\eta\sqrt{\Pr})\end{array}\right]$$

$$+\frac{Gct\sqrt{t}}{3(Sc-1)\sqrt{Sc}}\left[\begin{array}{l}\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2) - \frac{4}{\sqrt{\pi}}(1+\eta^2 Sc)\exp(-\eta^2 Sc) \\ -\eta\left(6+4\eta^2\right)erfc(\eta) + \eta\sqrt{Sc}\left(6+4\eta^2 Sc\right)erfc(\eta\sqrt{Sc})\end{array}\right]$$

(12)

where $\eta = \frac{y}{2\sqrt{t}}$, $erfc(x)$ being the complementary error function defined by

$$erfc(x) = 1 - erf(x),\ \ erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-\eta^2)d\eta$$

RESULTS AND DISCUSSION

For physical interpretation of the problem, numerical computations are carried out for different physical parameters $Gr, Gc, \Pr$ and Sc upon the nature of the flow and transport. Here the value of Pr is chosen as 0.71, which corresponds air. The values of Sc are chosen such that they represent water vapour (0.6) and Ammonia (0.78). In the present study we adopted the following default parameter values $Gr = 2.0$, $Gc = 2.0$, $\Pr = 0.71$, $Sc = 0.6$, $t = 0.2$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

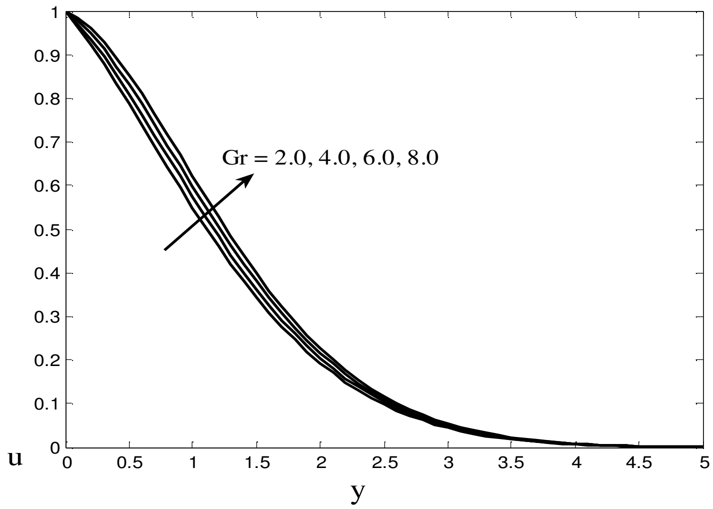

Figure 1. Velocity profiles for different values of Gr

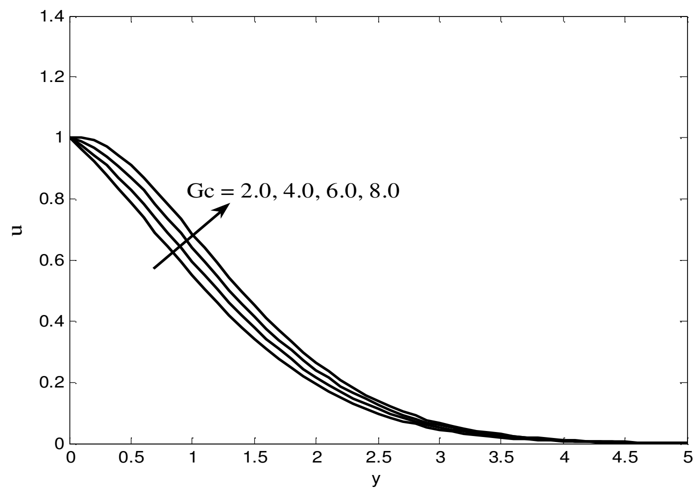

Figure 2. Velocity profiles for different values of Gc

The velocity profiles for different values of Gr are studied and presented in Figure 1. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. The flow is accelerated due to the enhancement in buoyancy force corresponding to an increase in the thermal Grashof number i.e., free convection effects. The positive values of *Gr* correspond to cooling of the plate by natural convection. Heat is therefore conducted away from the vertical plate into the fluid which increases the temperature and thereby enhances the buoyancy force. In addition, it is seen that the peak values of the velocity increases rapidly near the plate as thermal Grashof number increases and then decays smoothly to the free stream velocity.

Figure 2 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number Gc. The solutal Grashof number *Gc* defines the ratio of the species buoyancy force to the viscous hydrodynamic force. The solutal Grashof number Gc defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is noticed that the velocity increases with increasing values of the solutal Grashof number.

Figures 3 and 4 illustrate the velocity and temperature profiles for different values of Prandtl number Pr. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Figure 4, it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to

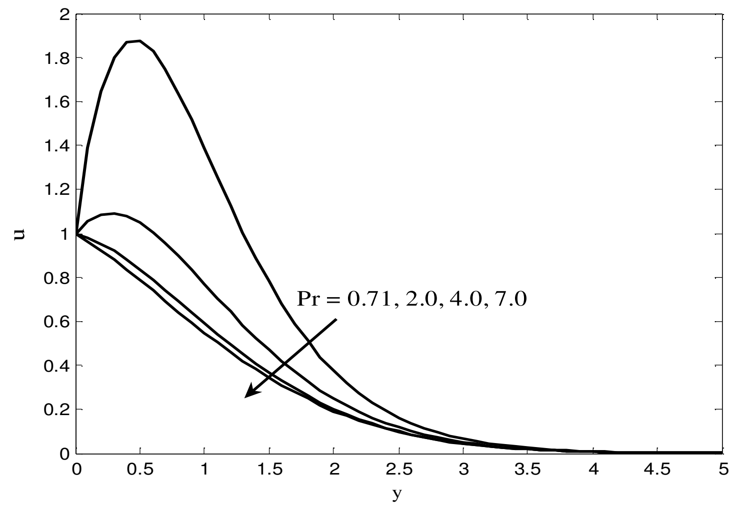

Figure 3. Velocity profiles for different values of $\Pr$

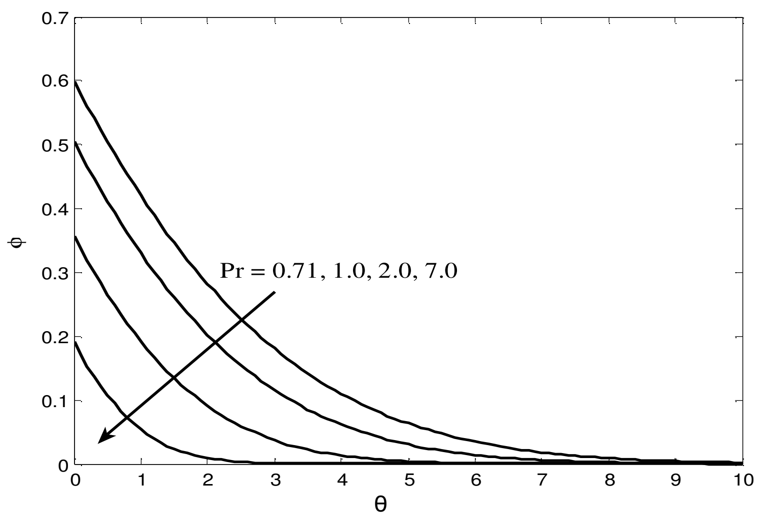

Figure 4. Temperature profiles for different values of $\Pr$

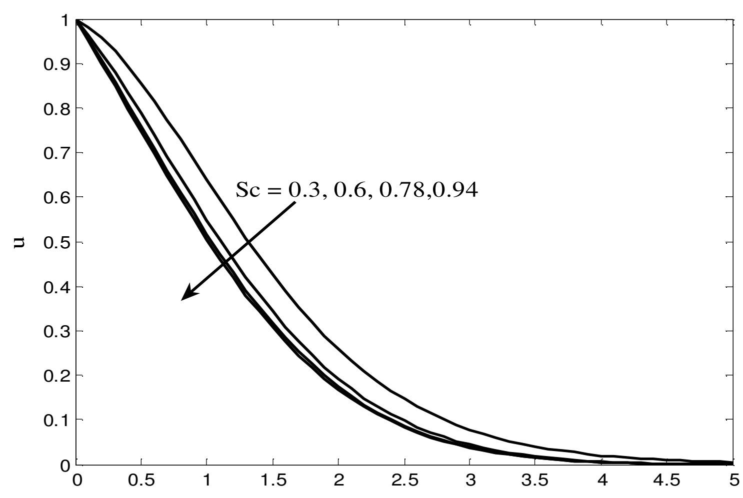

Figure 5. Velocity profiles for different values of Sc

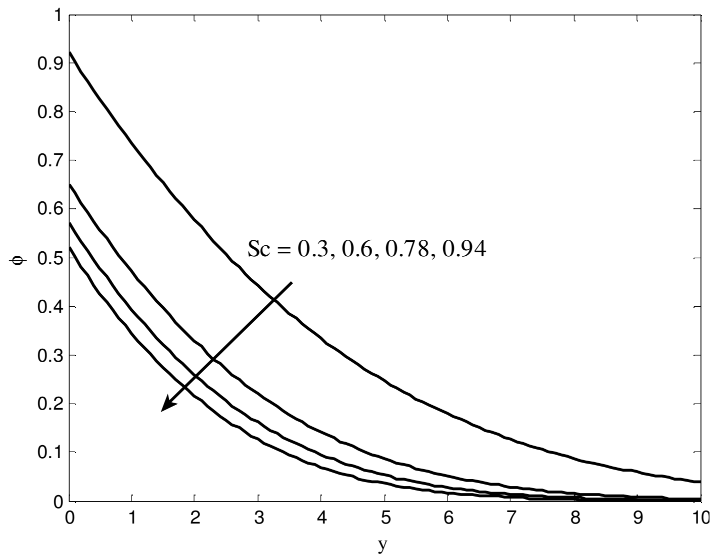

Figure 6. Concentration profiles for different values of Sc

diffuse away from the heated surface more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

For different values of the Schmidt number *Sc*, the velocity and concentration profiles are plotted in Figures 5 and 6 respectively. The Schmidt number *Sc* embodies the ratio of the momentum diffusivity to the mass (species) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass-transfer (concentration) boundary layer. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers, which is evident from Figures 5 and 6.

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