Educational Research (ISSN: 2141-5161) Vol. 3(11) pp. 860-865, November 2012
Available online@ http://www.interesjournals.org/ER

Full Length Research Paper

The variation concept: In the elementary school and college

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Abstract

In this paper is shown that the students have their first approach to the variation concept at the basic level; however, they only work upon directly proportional variation. When the student reaches, university level, it is this form of working upon the concept of variation that prevails, which represents an obstacle –among other things– to the analysis of the global behavior of different functions, a knowledge which is essential in Calculus courses. This situation motivated our design and the consequent application of activities which include the analysis of different functions, which were worked upon by students in the classroom through group discussions. 28 Mexican students at university level participated, since it is essential that those who enter university level are able to identify that the relationship between the variables of a problem is not always proportional. The activity being documented in this article, it was identified that the use of a value chart and the graphic exploration favored students' reflection.

Keywords: Variation, Primary School, College, activities.

INTRODUCTION

Students have their first approach to the study of variation at elementary education. In the Syllabuses and Course Contents of Elementary Education, In the case of Mexico, the notion of variation is included early in the topic named "processes of change" and it aims exclusively at developing in the student everything which concerns to directly proportional variation. More specifically, the development of the "processes of change", starts with simple situations in fourth grade and it is studied more deeply during the last two grades of elementary education. The lessons from elementary education textbooks in Mexico (SEP, 2011b), are composed by the reading, the making and the analysis of tables and graphs in which processes of variation should be registered and analyzed. The lessons end up with the notions of ratio and proportion, which are fundamental to the comprehension of various mathematical topics and for the solution of problems that take place in everyday life.

Piaget, 1978, points out that the subject can build the qualitative proportionality scheme when he comprehends that an increase in the independent variable produces the same result that a decrease in the dependent variable does. That is, when he or her comprehends that requires an element of compensation.

It may be inferred since Elementary Education, both independent and dependent variables are worked upon, focusing only on the idea of proportionality.

For those students in high school or in the early years of university, the variation theme is based upon the concept of function, it may be inferred from this that the connection between independent and dependent variables and the changes in the dependent variable, for changes in the independent variable, are essential elements for the knowledge of the change ratio and the construction and interpretation of graphs, which represent different phenomena. To relate the independent and dependent variables may be favored by both the numerical and graphic exploration using either pencil or paper or with the support of information and communication technologies such as the use of dynamic and simulation programs (Ruiz, 2010).

Background

Difficulties identified in high school students

Despite the importance of the study of variation, research related to this topic, such as the one by Ruiz (2010), Gravemeijer et al. (2007), Artigue (1991a), document that the students who have already taken the subject of Calculus express the following difficulties:

a) When working with variation situations they consider a directly proportional variation between the variables of the problem.

b) They do not manage to relate the representation records such as the use of the table, with the algebraic expression and the graph, when solving a problem.

c) They employ the rule of three as their first option, when solving variation problems and they do not always succeed in its use.

Research Problem

In accordance with the difficulties mentioned in the paragraph above, this article makes reference to the tendency shown by students when they work considering a directly proportional variation between the variables of the problem (through the use of the rule of three) as the first strategy when solving variation problems.

Research Hypothesis

The use of multiple representations and the relationship between them are considered elements which may contribute that high school students modify their conception that variables present in a problem are related in a directly proportional way.

Objectives of Study

This paper has two objectives, the first one show that students at university level consider, in their early explorations, that the variables involved in the problems are related through a proportional variation. Emphasizing the idea that students at university level show a similar thought to the one which is developed by students at elementary level. The second objective is to show an activity in which the students manage to overcome the belief that everything varies in a proportional way. Through the use of the different modes or registers of representation.

Theoretical Framework

The theoretical framework for the research is divided in two sections. The first one makes reference to the study of variation at elementary level and the second section studies what concerns to the representation of functions at high school and university level:

A) Study of the variation concept at elementary level.

B) Study of the variation concept at high school and university level.

Study of the concept of variation at elementary level

Piage, 1978, states that between the ages of 11 and 12 it is possible to see in the subject the presence of the notion of proportion in different fields, such as: spatial proportions (similar shapes), the probabilities, etc. Piaget, 1978 and Streefland, 1991 and 1993, stress that in order for the student to develop correctly the notion of directly proportional variation, it is necessary to begin with the notions of expansion and reduction following the idea of the photocopy or the scaled drawing, assuming that the student at a very early age manages to recognize what is proportional.

It may be inferred that since Elementary Education both independent and dependent variables are worked upon, focusing only on the idea of proportionality.

On the other hand, studies conducted by Hart, 1988, point out that the notion of directly proportional variation is worked upon by students through the use of the rule of three, that is to say in a mechanical way, because it is thus taught by most teachers, without their promoting in the student a proportional qualitative thought before the quantitative one in order to assign meaning to the use of proportional variation.

In regard to this, a previous study of an exploratory character (Ruiz, 1997a, 1997b and 1997c; Ruiz and Valdemoros, 2002), in which they worked with sixth grade elementary students and teachers both from elementary and junior high school, was an indicator of the use of the simple rule of 3 as the only tool to solve problems of lost value in its solution, that in the case of the teachers , the use of this algorithm was successful, which did not happen with students, adding the fact that there was no manifestation of understanding of its use. This previous study allowed us to see the importance of education when it is given under certain kind of educational organization when it comes to solving problems of lost value through the use of an algorithm. Hart, 1988, points out that if the teaching does not introduce the rule of three, so, it won't appear by itself.

Both, teachers and students reduced the treatment of ratios and proportions to a solution of the kind of problems mentioned above, through the use of the rule of three. It was verified, the little weight that the teaching of a committed election has when it comes to ratios and proportions, being that simple proportional variation is included in sixth grade course contents, in most of the

modules which compose it.

Study of the variation concept at high school and university level

In the literature related to the study of variation a fundamental recommendation is expressed by Hauger, 2006 and Nemirovsky, 2006. Indicates the convenience of developing three kinds of knowledge of change ratio in order for a student to identify the behavior of a function: global change ratio, in one interval and punctual. The global change ratio is concomitant to the general properties of a function, like its monotony. The knowledge of change ratio, in one interval, refers to the change of the dependent variable for different intervals among which the independent variable is found. Punctual change ratio (instant) has to do with the speed at which the dependent variable changes with regard to one value of the independent variable. Hauger, 2006 points out that these three kinds of knowledge of change ratio could be examined using different representation of the functions, including graphs, grids, equations and verbal descriptions.

Nemirovsky, 2006, on the other hand, considers that it is important to include in the work of students both a global and local analysis of functions. According to the author, it is learning activities which enable that a subject starts to resort to alternative approaches which allow them to know the information given by the graph of a function and relate it with the graph of its derivative. Such way of working is known as variational approach.

Nemirovsky emphasizes that an approach of this kind includes two ways of work: to analyze the function locally and globally, he also points out that when one student does not have enough experience to describe the global and local behavior of a function, he applies a strategy of similarity between the graphs (the graph for the function and the graph of its derivative).

Similarly, Nemirovsky, 2006 points out :

"[…] *The construction of a variational approach is a complex process which involves coordinating several different pieces of knowledge. It takes place throughout time and includes steps forwards and backwards. A student often seems to have constructed elements of a variational approach, but when it comes to the solution of a more complex problem he resorts to use non variational techniques such as the ones of similarity.* […]" (p. 21).

From the work show by Hauger, 2006 and nemirovsky, 2006, it can be inferred that for the study of the phenomena which involve change or variation, it is advisable that students explore problems in why they identify, interpret, and analyze the local and global behaviors of the functions.

About relation to the representations, (Villegas et al., 2009, Goldemberg, 1995, Tall, 1996, and Kieran, 2003), point out that representations play an fundamental role in the mathematical thinking as they favor the comprehension of the mathematical concepts and stimulate the development of flexible thinking in the solution of problems.

METHOD

Subject

The study was carried out with 28 university level students in the first course of mathematics. 28 students, whose ages range between 18 and 19 years, participated, they come from a technological high school, and they have already taken Algebra courses, Trigonometry, Analytic Geometry and Differential and Integral Calculus.

Instruments

The research, from which partial results are reported here, is located within a qualitative research paradigm.

The instruments employed during the collection of data were:

a) Written reports produced individually.
b) Written reports produced by each pair of students.
c) Video and audio recordings from the work of students.
e) Reports produced by the teacher- researcher.

Procedure

The ideas developed in the theoretical referents served as support to design and apply three activities, from which one is reported, in which students identified, interpreted and analyzed the global and local behavior of functions. These elements were considered to be important for the students, to analyze the way in which functions vary, where the concept of variation is found immerse, which is the fundamental aspect in the study of Calculus.

The number of sessions for each activity varied, according to the time students needed for the exploration and analysis of information and the discussion of the important mathematical ideas involved in the solution process.

The elements which conduct the analysis are:

1. To Identify the way in which students represent and explain situations which involve change or variation.
2. To know how they relate the variables involved in situations which represent change or variation.
3. To document the way in which they shift from a punctual or local analysis of the function behavior which represents the phenomenon to a global

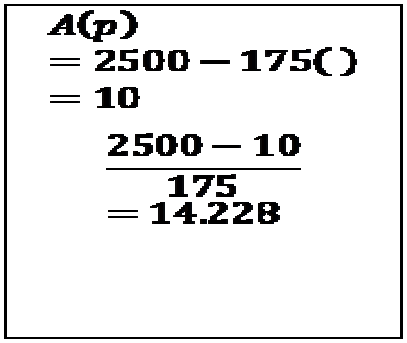

Figure 1. Antonio and Manuel's answer to question b.

explanation.

4. To know the way in which they construct functional relationships which make sense to the correspondence between the values of the domain and the range of a function (relationship between variables).

ACTIVITY AND RESULTS

It is important to emphasize that one of the objectives of this paper is to document that one first ways in which students work is to consider that the variables relate themselves proportionally, therefore the analysis of the data and the discussion will be made in this direction, which does not minimize the importance to the development and the variation concept.

In this paper the work of three students in one activity is described. The analysis of the written reports, of video recordings and interviews provided elements to know about the processes of the building of relationships between representations which the students followed, when working on the activities. The extracts being presented here refer to the work of Antonio, Manuel and Rodrigo, who were considered representative samples of the group.

Activity; Attending a show

Suppose that in a market research it has been found that the number of people who attend a show depends on its cost. This situation can be represented by the following formula A(p) = 2500 – 175p where,

A = represents the number of people who attended the theater.

p = represents the price of the ticket.

Students were asked to answer the questions:

a) Which is the maximum number of people who can attend the show?

b) Which is the maximum price which can be charged for the entrance to the show in order to guarantee the entrance of at least 10 people?

c) What will be the entrance to the show if the price of the ticket is $10?

d) What will be the attendance if the entrance is free?

At the beginning Antonio and Manuel worked individually, they carried out a punctual analysis of the function, they substituted different values of the price of the ticket in the given formula and with these values they answered question *c* correctly. In order to answer question *b* they worked out the *p* variable and substituted the number of people who attended the show (Figure 1), with this, they answered the question which is the price… to have the attendance of at least…?

The statements made by Manuel confirmed what was mentioned above:

At this point I worked out and assigned a value of 10 people in order to substitute and draw the conclusion that $14 should be the minimum price for an attendance of 10 people.

This way of work coincides with what Nemirovsky, 2006 reported, he points out that students can carry out a punctual analysis of the functions –they substitute values when a formula is provided– but this does not guarantee that they understand the global behavior of a function and the relationship between the variables involved in the problem.

Probably the answer of Antonio and Manuel was influenced by the fact that they only substituted the values in the algebraic expression provided, which could make it difficult to analyze the global behavior of the function.

Working on the assumption, the teacher-researcher considered that the exploration of a table or a graph could contribute that the student carry out a global analysis of the behavior of the function and he had an intervention in this direction. It was asked that students build a table and a graph and they were asked to answer the following questions:

Which are the variables involved in this activity?
Calculate the attendance if the price of the ticket is $0, $2, $8, $5, $14 and $12.
Order the results obtained in the previous point in a table like the following:

Price	Attendance

Locate within a Cartesian plane the points with coordinates (price, attendance).

From the graph, identify the change in the number of people who attend the event when the price of the ticket changes from $0 to $1. Identify the change in the number of people who attend the event when the price of the ticket changes from $1 a $2.

If the price of the ticket increases what happens to the number of people present in the event?

On the other hand, Rodrigo's work showed more clearly his idea of directly proportional variation. As it is stated by Ruiz, 1997a, 1997b, 1997c, and Ruiz and Valdemoros, 2002. The students of elementary education and junior high school, use the simple rule of 3 as the only tool to solve problems of lost value, this way of working was identified in the students who participated in this research, as it was the case or Rodrigo, who stated the following in his report:

"I thought that everything was going to work with a simple rule of three, but it didn't. After that, with the questions the teacher posed I started to perceive a different perspective of the problem and understood that only p and A were going to vary."

After identifying that this way of working appeared in most students, the researcher took part in order to direct his reflection and favor the shift from a punctual analysis to a global one which would allow them to answer the questions related to the behavior of the function in an interval. In order to answer question *b* it is necessary to consider the attendance of 10 or more people. The interaction of the researcher with Rodrigo gives evidence that the reflection that the student made and shows that the student changes from a punctual analysis to a global one.

Researcher: What does it mean that at least 10 people attend?
Rodrigo: It would be from 10 people onwards.

The intervention of Rodrigo brought about that other classmates of the group take part giving examples of the price of the ticket for 10, 11, and 12… people. The students identified the appropriateness of making a table with an increase of 10 people from 0 until 2500. Rodrigo, who had identified the global behavior of the function, directed the discussion; he verbally expressed the meaning of "at least 10" and answered correctly the remaining questions. With this the session finished.

In this activity it was identified that when students solve problems in which they have to formulate a mathematical model one first strategy they employ is to consider that the quantities change in a proportional way. Even when the formula is provided to the students, they substitute values in the formula and assume that the relationship between the variables is linear, without carrying out an analysis of the general behavior of the function.

DISCUSSION AND CONCLUSIONS

Even when in elementary education in Mexico (2011a, 2011b), the making and the analysis of tables in which processes of variation should be registered and analyzed are included, the results from this research show a tendency –in the students that participate– to apply the rule of 3 as the first strategy to solve problems. This can make it difficult for students to carry out a global analysis of the behavior of the functions which makes it possible that they identify the way in which functions vary, which is an essential aspect in Calculus courses.

In the documented activity it was identified that the use of a table of values and the graphic exploration favored the reflection on behalf of the students in order to organize their knowledge, relate the variables of the problem, modify their local analysis to a global analysis of the behavior of the function. According to Nemirovsky, 2006, it is important for a student to analyze the behavior of a function in two levels: analyze the function locally and globally.

In order to achieve that the student carries out this ways of work, the role of the teacher is fundamental, it is the teacher who should direct students' reflection in order to achieve that they shift from a local analysis to a global one. The evidence found show how important it is that the teacher identifies the moment in which the tabular and graphic explorations can contribute that the student makes a deeper reflection, and with this, he achieves to better comprehend the behavior of a phenomenon.

The analysis of the values in the table and the graph in the activity contributed to the carrying out of the comparison of the variation of the functions used in such activity.

Even though it is not possible to generalize about the processes which students follow to enhance their knowledge in function behavior, the results from this research give evidence of the benefits of including in the work of the students the analysis of different kinds of functions, through their graphical, tubular and algebraic exploration. The construction of the variation concept is a process that takes time and has steps backwards that is why it should be reinforced through activities that promote the conscious reflection on behalf of students at different educational levels.

REFERENCES

Artigue M (1991). Analysis. In D. Tall (De.). *Advanced Mathematical Thinking*. The Netherlands: Kluwer Academic Publishers, Mathematics Education Library. 11, 167–198

Elena FR (1997ª). Resolución de problemas a nivel primaria haciendo uso de la calculadora Math Explorer. *Resúmenes de la Reunión Latinoamericana de Matemática Educativa. Comité Latinoamericano de Matemática Educativa.* . 245.

Elena FR (1997b). Taller de Resolución de problemas a nivel medio, haciendo uso de la calculadora Math Explorer Plus. *Memoria del XVI Congreso Nacional de la Enseñanza de las Matemáticas* 17-18. Asociación Nacional de Profesores de Matemáticas. Escuela Normal Superior del Estado de México.

Elena FR (1997c). Uso de las calculadoras Math Explorer y TI-92 en la Resolución de Problemas: Una experiencia con profesores de los niveles básico y medio. *Memorias del Seminario Nacional de Calculadoras y Computadoras en Educación Matemática.* . 25-35.

Elena FR (2010). Estrategias didácticas con uso de la tecnología en la contribución del desarrollo de competencias matemáticas en el nivel superior. *Ide@as CONCYTEG* Vol 5(61). 731-744

Elena FR, Marta V (2002) .Concepts of ratio and proportion in basic level students: case of study. In:D. Mewborn, P. Sztajn, E. White, H. Wiegel, R. Bryant & K. Nooney (Eds.). *Proceedings of the 24th Annual Meeting of the North American Chapter of the International Group For the Psychology of Mathematics Education:* Vol. 4 2002. pp. 1651-1657. Columbus, OH: Eric Clearinghouse for Science, Mathematics and Environmental Education.

George H (2006). *Rate of change knowledge in high school and college students*. Paper presented at the Annual Meeting of the American Educational Research Association. San Francisco, CA. Recovered on the 18 of July 2006, from: http://www.eric.ed.gov:80/ERICWebPortal/

Goldenberg EP (1995). Multiple Representations: A Vehicle for Understanding, Perkins D. N. et al (Eds.), Oxford University Press, 155-171.

Gravemeijer K, Cobb P, Yackel E, McClain K (Eds.) (2006). *Symbolising and communicating in mathematics classrooms. Perspectives on discourse, tools, and instructional design*. Mahwah, New Jersey: Lawrence Erlbaum Associates Publishers. 225-274

Jean P (1978). Las operaciones intelectuales y su desarrollo. In J. Delval (Comp.), *Lecturas en Psicología del Niño, I* . 70-119. Madrid: Alianza Editorial.

Katlen H (1988). Ratio and proportion. In J. Hiebert and M. Behr (Eds.), *Concepts and operations in the Middle Grades,* Reston, Virginia: National Council of Teachers of Mathematics *2,* 198-219.

Kieran C (2003). Functions, Graphing and Technology: Integrating Research on Learning and Instruction in Integrating Research in the Graphical Representation of Functions, T. Carpenter, E. Fennema and T. Romberg (Eds.), Erlbaum Hillsdale, N.J. 189-237.

Lee S (1991). *Fractions in realistic mathematics education*. P.h. D. thesis published by Kluwer Academic Publishers. 1991 46-134.

Lee S (1993). The design of a mathematics course a theoretical reflection. *Educational Studies in Mathematics.* Vol. 25, 109-135.

Nemirovsky R (2006). *Students´ Tendency to assume resemblances between a function and its derivative*. Reports-Research/Technical (143). Recovered on the 16 of mayo, from: http://edres.org/eric/ED351193.htm

Secretaría de Educación Pública (2011ª). *Plan de programas de estudio. Educación Básica. Primaria.* Dirección General de Materiales y Métodos Educativos de la Subsecretaría de Educación Básica y Normal. . México D. F.: SEP.

Secretaría de Educación Pública (2011b). *Matemáticas. Sexto grado.* Comisión Nacional de los Libros de Texto Gratuitos.México D. F.: SEP.

Tall D (1996). Functions and Calculus Education . En: A. J. Bishop et al (eds.), *International Handbook of Mathematics* . First Edition Kluwer Academic Publishers, Netherlands:289-325.

Villegas JL, Castro E, y Gutiérrez J (2009). Representaciones en Resolución de Problemas: Un estudio de caso con problemas de optimización. *Revista Electrónica de Investigación Psicoeducativa. No. 17 Vol. 7(1)*, 279-308.