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Full Length Research Paper

The mathematical law of chaotic dynamics applied to cardiac arrhythmias

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Abstract

From dynamical systems theory and a mathematical deduction of Box-Counting equation, a chaotic exponential law that objectively differentiates normal cardiac dynamics from pathological ones, and the evolution between these states was inferred. 25 Holters were taken from a database: 16 with arrhythmias and 9 clinically diagnosed as normal but with various symptoms or indications. A simulation was made of heart rate to construct the attractors of the dynamics, and calculate the occupation of spaces in two grids and its fractal dimension. The cases diagnosed as normal, but with different indications or symptoms, presented a number of occupied spaces with the grid Kp between 84 and 192, and between 21 and 49 for the kg grid, with a mathematical diagnosis of evolution to abnormality. For arrhythmic patients, number of spaces occupied by attractors in small grid varied between 47 and 165 and on the big screen between 14 and 52, with a mathematical diagnosis of progression to abnormality or in acute disease. This study revealed the general applicability of this methodology for evaluating the dynamics of different types of cardiac arrhythmia and for detecting slight changes of dynamics, which are not clinically identified as pathological.

Keywords: Law, chaos, fractal, cardiac dynamics, Holter, diagnosis.

INTRODUCTION

Arrhythmias are responsible for about 50% of cardiovascular deaths (Gaziano and Gaziano, 2011). Presented in the form of palpitations, are a very common cause for consultation in primary care. The first goal of the physician is to establish whether those palpitations are due to a cardiac arrhythmia or not, and for this reason it is essential the electrocardiographic documentation of an episode (Baquero *et al.,* 2010), for example, through a Holter study. Once detected, the treatment will vary depending on parameters such as the type of arrhythmia in question, the severity of symptoms and the presence or absence of underlying structural heart disease. Like any other disease, treatment should cover two objectives: to relieve symptoms and prolong survival (Baquero *et al.,* 2010).

Cardiac arrhythmia is a frequent event, and often confounds the primary care physician. In acute and symptomatic episodes, usually doubt arises about which drug to use in afraid of worsening the case. On the other hand, it is often to incidentally detect different types of arrhythmias in routine electrocardiograms of asymptomatic patients; the doubts in this case usually

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arise about to whether or not it is necessary to treat them. Identifying the type of rhythm disorder in an electrocardiogram is the first step to address these events (Baquero *et al.,* 2010).

The theory of dynamic systems describes the behavior of the variables of the system, which define its status and evolution. For this, abstract spaces were developed, built with dynamic variables of the system, such as return maps, from which one can determine whether the system is predictable or unpredictable through three kinds of attractors: fixed point, limit cycle and strange attractor (Devaney, 1992; Peitgen, 1992a). The fixed point attractor discloses a system which tends to a state where its dynamic variables takes a constant value, as the case of a physical pendulum which by friction comes to an idle state. The limit cycle attractor is a system in which the dynamic variables periodically take the same values, as in the case of an ideal pendulum, which remains in perpetual periodic motion. Finally, the strange attractor describes a system in which there is no a trend towards a particular state, nor a periodic behavior with respect to a set of states. From this construction, chaos is conceived as dynamic as the other systems but fundamentally unpredictable and dependent on initial conditions (Crutchfield *et al.,* 1990).

One of the mathematical conditions of a strange attractor is its irregularity, which can be characterized mathematically by the fractal dimension, a non-dimensional measure from fractal geometry (Mandelbrot, 2000 a, b) developed by Benoît Mandelbrot. The method for calculating fractal dimensions depends on the type of object being measured; in this way the Hausdorff dimension can be used to abstract objects (Peitgen, 1992b), whereas the characterization of an object with overlap in its parts –“wild-fractals”- is calculated through the method of box-counting, generally used in measurement of natural objects (Peitgen, 1992c). Another type of fractals, the statistical fractals, are based on Zipf-Mandelbrot law and have allowed the development of a diagnostic objective and reproducible method as to evidence the self-organization of language (Mandelbrot, 2000c), immune system (Burgos, 1996) specifically allergies (Rodríguez, 2005), and the application of this law to the analysis of fetal monitoring (Rodríguez, 2006b). Other methods worked by the authors of this study include frequency analysis in Fourier Transform and similar applications (Narváez and Jaramillo, 2004).

Fractal geometry has been used in studies on cancer (Luzi *et al.,* 1999; Gazit *et al.,* 1997), and has allowed to establish diagnostic differences of clinical and experimental relevance between normal and pathologic ventriculograms (Rodríguez et al., 2006c; Rodríguez *et al.,* 2012b) as well as the fractal evaluation of left coronary branch in the presence and absence of arterial occlusive mild, moderate and severe disease (Rodríguez *et al.,* 2004; Rodríguez *et al.,* 2012d).

Based on the theory of dynamical systems applied to physiology, Goldberger suggested that a pathologic system have high periodicity or excessive randomness, while a healthy system is associated with a more moderate fluctuation without reaching such extremes (Goldberger *et al.,* 2002). With those kind of methodologies, from calculations of fractal dimensions of RR intervals in patients with acute myocardial infarction, better risk predictors than those conventionally applied in the clinic have been achieved (Huikuri *et al.,* 2000).

Previously, an exponential law of clinic application was performed; it allows to deduct the total of possible discrete cardiac chaotic attractors. Based on this law is possible to differentiate normality, acute disease and evolution between them (Rodríguez, 2011b). That work evaluates the space occupation of cardiac attractors in the general space of Box Counting, establishing that the occupied spaces values of kp in the order of tens correspond to acute cases’ Holters, whilst the normal Holters’ values include values greater than 200; and the evolution cases present values between those two limits. The graphical representation of the occupied-space proportions' values (Kp/kg) respect to the fractal dimension shows that the variables have an exponential behavior.

The purpose of the present work is to apply the previously developed methodology, based on the exponential law (Rodríguez, 2011b), to 25 Holters from healthy or arrhythmic patients in order to establish the methodologies’ ability to quantify the severity level of these dynamics.

Definitions

Fractal

From Latin *fractus*. Irregular as a noun, irregularity as an adjective. For wild fractals usually the Box-counting method is used in order to evaluate the fractal dimension.

Return Map

specific attractor that plots in a space of two or more dimensions, the dynamics of a system, locating ordered pairs of values from a dynamic variable and consecutive in time.

Box-Counting Fractal Dimension

$$D = \frac{LogN\ (2^{-(K+1)}) - LogN\ (2^{-K})}{Log\ 2^{k+1} - Log\ 2^{k}} = Log_{\,2}\frac{N(2^{-(k+1)})}{N(2^{-k})}$$

equation 1

D: fractal dimension.

N: Number of boxes occupied by the object.

K: Grade of partition in the grid.

METHODS

Population

25 Holters were studied, 16 from patients with various kinds of cardiac arrhythmias, and 9 patients diagnosed as normal, but with indications of symptoms such as syncope, heart palpitations, hypertension, chest pain being studied or tachycardia being studied, from the Hospital Universitario San Vicente De Paul, Medellín Colombia. The total value of beats for each hour was taken, as well as the minimum, maximum and intermediate values of the heart rate, for 21 hours for each test. A simulation sequence of heart rates in the range defined was made and an attractor dynamics of each was built in the return map, which was plotted in a two dimensional space facing against one frequency with the next one.

Then the fractal dimension was evaluated with the box-counting method (see definitions) by quantifying the boxes occupied by the attractor, using two grids, with 5 and 10 beats / minute.

Following the methodology developed by Rodriguez (2011b), when the box counting equation is assessed with two grids in which one is twice the other, the equation can be rewritten as:

$$D = \log_2 \frac{K_p}{K_g}$$

Where K_p is the number of occupied boxes with the small grid, and K_g is the number occupied boxes with the big grid.

From this equation the term that evaluates the occupied boxes from the small grid is cleared:

$$K_p = K_g 2^{DF}$$

Where

Kp: occupied boxes by the attractor on the small screen.

Kg: occupied boxes by the attractor in the big grid.

DF: fractal dimension.

Solving for the number of occupied boxes by the attractor within the large grid, it is obtained:

$$K_g = \frac{K_p}{2^{DF}}$$

The results obtained were compared with conventional diagnostics, in order to confirm the clinical applicability of the methodology and the differences and / or agreements with the conventional method.

Statistical Analysis of the cases with arrhythmia

Starting from the 16 Holters conventionally diagnosed with arrhythmia, and taking the Holter's clinical diagnosis as a gold standard, they were compared against the physical-mathematical methodology's results, calculating sensitivity and specificity values. Later in order to determine the correlation between physical-mathematical diagnosis and conventional diagnosis, Kappa coefficient was calculated by the following formula:

$$K = \frac{Co - Ca}{To - Ca}$$

Where:

Co: observed number of matches, ie number of patients with the same diagnosis according to the proposed new methodology and the Gold Standard.

To: the entirety of observations.

Ca: Random concordances, which are calculated according to the following formula:

$$Ca = [(f_1 x C_1)/To] + [(f_2 x C_2)/To]$$

Where f_1 is the number of patients with mathematical values within normal limits, C_1 is the number of patients clinically diagnosed within normality, f_2 is the number of patients with mathematical values associated to arrhythmia, C_2 is the number of patients diagnosed clinically with arrhythmia and To is the total number of normal and arrhythmia cases.

RESULTS

The fractal dimensions of the analyzed 25 attractors oscillated between 1.5459 and 2. The number of occupied spaces with the first grid, Kp, for the attractors, oscillated between 47 and 192; with the second grid, Kg, between 14 and 52 (See table 1).

Of the 25 evaluated attractors, 7 presented occupied values under or equal to 73 in the Kp occupied spaces related to acute cases, and the remaining values between 84 and 192, associated to evolution, according to the work previously done (Rodríguez, 2011b), (See Table 1).

The cases diagnosed as normal, but with indications such as syncope, heart palpitations, hypertension, chest pain being studied or tachycardia being studied, presented a number of occupied spaces with the grid Kp between 84 and 192, and between 21 and 49 for the kg grid. The mathematical diagnoses of these cases correspond to evolution between normality and disease. None of these cases was mathematically diagnosed with acute disease. The results demonstrate the ability of the methodology to detect slight changes of dynamics, which are not clinically identified as pathological (See table 1).

The cases clinically diagnosed with arrhythmia presented values of occupied spaces with the grid Kp between 165 and 47, and with the second grid, Kg, between 52 and 14. After observing these cases, it is

Table 1. Number of occupied spaces (Kp, Kg) and fractal dimension (DF) of chaotic attractors of the patients studied. The shaded rows correspond to cases with clinical diagnosis of normal, but with signs or symptoms as syncope, heart palpitations, hypertension, chest pain being studied or tachycardia being studied

No.	Conclusions	VE	SVE	Indication	Age	K_p	K_g	DF	2^df	DX
1	Sinus rhythm, normal AV conduction, with periods of low atrial rhythm. Occasional isolated ventricular and supraventricular ectopic beats were seen. The ST segment was unchanged. Good heart rate variability. Experienced no symptoms.	29	117	Syncope	89	73	25	1,55	2,92	ACUTE
2	Sinus rhythm without alterations in the origin or impulse conduction. The ST segment was unchanged. Good heart rate variability. Experienced no symptoms.	21	8	Discard Embolism	79	161	47	1,78	3,43	Evolution
3	Sinus rhythm without alterations in the origin or impulse conduction. The ST segment was unchanged. Good heart rate variability. Experienced no symptoms.	1	0	Syncope	24	143	48	1,57	2,98	Evolution
4	Sinus rhythm, normal A-V conduction. Very frequent, monomorphic ventricular ectopic beats with long periods of bigeminy were observed. The ST segment was unchanged. Heart rate variability is not interpretable. When symptoms were experienced the described ectopic beats were observed, with no difference from the ones observed when patient was asymptomatic.	6920	2	Tachycardia	50	84	23	1,87	3,65	Evolution
5	Sinus rhythm. Normal A-V conduction. Isolated, monomorphic ventricular ectopic beats were observed. The ST segment was unchanged. Decreased heart rate variability. Experienced no symptoms.	71	1	Chest Pain being studied	71	96	27	1,83	3,56	Evolution
6	Sinus rhythm without alterations in the origin or impulse conduction. In an emotional moment when patient referred heart palpitations, sinus tachycardia was observed. The ST segment was unchanged. Slight decrease in heart rate variability.	0	0	Syncope	42	88	24	1,87	3,66	Evolution
7	Sinus rhythm, normal A-V conduction. Occasional supraventricular ectopic beats. Short episode of supraventricular tachycardia. The ST segment was unchanged. Good heart rate variability. Experienced no symptoms.	15	18	Hypertension	70	120	32	1,91	3,75	Evolution
8	Patient experienced atrial fibrillation during most of the study with some episodes of sinus rhythm. Some supraventricular ectopic beats were observed with duplets and triplets, and an episode of pacemaker migration. Some isolated ventricular ectopic beats of different morphology were observed. The ST segment was unchanged. Heart rate variability is uninterpretable. Experienced no symptoms.	181	6112	Heart palpitations being studied	75	64	16	2,00	4,00	ACUTE

Table 1. Continuation

9	Patient was in atrial flutter almost all the time with 2:1 block, with exceptional periods of more advanced blocks.	3	2747	IAC – Atrial Flutter	36	119	36	1,72	3,31	Evolution
10	Sinus rhythm all the time, no alterations in the origin or impulse conduction. The ST segment was unchanged, heart rate variability satisfactory. Experienced no symptoms. Completely normal Holter.	0	0	Tachy-arrhythmias	21	192	49	1,97	3,92	Evolution
11	Sinus rhythm, normal A-V conduction. Very occasional isolated monomorphic ventricular ectopic beats were appreciated. Some supraventricular ectopic beats and a short period of atrial tachycardia were observed. Episodes of dyspnea associated with sinus tachycardia. ST segment was unchanged. Good heart rate variability.	5	12	Chest Pain. Tachyarrhythmia	50	165	52	1,67	3,17	Evolution
12	Sinus rhythm without alterations in the origin or impulse conduction. In an emotional moment when patient referred heart palpitations, sinus tachycardia was observed. The ST segment was unchanged. Slight decrease in heart rate variability.	0	0	Heart Palpitations.	31	84	21	2,00	4,00	Evolution
13	Sinus rhythm, PR of 120 ms, with several episodes of sinus arrest, the largest of 2,859 ms. Some isolated supraventricular and ventricular ectopic beats were seen. The ST segment was unchanged. Normal heart rate variability. Experienced no symptoms.	3	1	Vertigo	46	129	42	1,62	3,07	Evolution
14	Sinus rhythm without alterations in the origin or impulse conduction. The ST segment was unchanged, with periods of prolonged QTc. Good heart rate variability. When patient was symptomatic no special changes were observed.	0	0	Syncope	34	119	40	1,57	2,98	Evolution
15	Sinus rhythm all the time, with an average heart rate of 88/min that increased properly with exercise. The maximum was 146 at 12 noon, moment in which there was no activity registered, but patient remained asymptomatic. The ST segment was unchanged. Normal heart rate variability.	0	0	Tachycardia being studied	37	141	47	1,58	3	Evolution
16	Patient experienced atrial fibrillation throughout the register and a significant increase in the ventricular automaticity with different morphologies of ectopic beats, with duplets and triplets. ST segment was unchanged. Heart rate variability was not evaluable. Experienced no symptoms.	8964	7157	Congestive Heart Failure	73	110	35	1,65	3,14	Evolution
17	Sinus rhythm without alterations in the origin or impulse conduction. The ST segment was unchanged. Good heart rate variability. Experienced no symptoms.	0	1	Chest Pain being studied	51	105	32	1,71	3,28	Evolution

Table 1. Continuation

18	Sinus rhythm with episodes of asymptomatic daytime sinus bradycardia, normal A-V conduction. Presence of supraventricular ectopic beats with episodes of non-sustained atrial tachycardia. There are times when the ST depresses horizontally. Periods of prolonged QTc. Good heart rate variability. Experienced no symptoms.	0	407	Bradycardia	32	65	19	1,77	3,42	ACUTE
19	Sinus rhythm. Normal A-V conduction. Permanent complete right bundle-branch block. Occasional isolated monomorphic ventricular ectopic beats were observed. Very frequent supraventricular ectopic beats were seen, with short periods of supraventricular tachycardia, likely paroxysmal atrial fibrillation. The changes of the right bundle-branch block were unchanged. Heart rate variability is uninterpretable. Experienced no symptoms.	912	5393	Cardiac Arrhythmia	62	161	51	1,66	3,16	Evolution
20	Atrial fibrillation with average ventricular rate of 62/min. Occasional ventricular ectopic beats. The ST segment was unchanged. Heart rate variability is uninterpretable. Experienced no symptoms.	203	9734	AF- CVA	92	85	26	1,71	3,27	Evolution
21	Sinus rhythm, normal A-V conduction. Significant increase in the supraventricular automaticity with frequent supraventricular ectopic beats (a quarter of all depolarizations), with duplets, triplets, and very frequent episodes of multifocal atrial tachycardia. Some wide complexes that meet aberrancy standards are appreciated. The ST segment was unchanged. Heart rate variability is uninterpretable. Experienced no symptoms.	4518	19930	CVD, Intermittent AF	71	56	14	2,00	4,00	ACUTE
22	Sinus rhythm without alterations in the origin or impulse conduction. The ST segment was unchanged. Good heart rate variability. Experienced no symptoms.	1	2	Unspecified Cerebrovascular Disease	71	86	23	1,90	3,74	Evolution
23	Sinus rhythm, A-V conduction delay (1st degree AV block). Occasional ventricular monomorphic ectopic beats were seen, as well as isolated ventricular bigeminy. Very occasional supraventricular ectopic beats with two short episodes of atrial tachycardia. The ST segment was unchanged. Decreased heart rate variability. Experienced no symptoms.	145	161	Three-vessel Disease – Second Degree AV Block	69	49	16	1,61	3,06	ACUTE
24	Sinus rhythm, normal A-V conduction. Very frequent ventricular ectopic beats (25% of all depolarizations) with a predominant morphology and frequent duplets were appreciated. Occasional supraventricular ectopic beats with short periods of atrial tachycardia. The ST segment was unchanged. Experienced no symptoms.	17316	48	VT	55	47	16	1,55	2,94	ACUTE

Table 1. Continuation

25	Comes in and out from sinus rhythm to fibrillo-flutter throughout the study. While in sinus, the PR was normal, with frequent supraventricular ectopic beats, even in duplets and triplets, with frequent ventricular ectopic beats with duplets and triplets as well. While in AF rhythm, there are some wide complexes with Ashman phenomenon characteristics. Patient had short episodes of multifocal atrial tachycardia. ST segment was unchanged. Heart rate variability was uninterpretable. In the symptomatic moments, no particular changes were appreciated. The EKG shows moments of AF, and when out of it, there are very long pauses.	6940	16676	Coronary Disease. SND	69	64	20	1,68	3,20	ACUTE

found that all the clinically diagnosed cases as arrhythmia, presented a mathematical diagnosis of evolution between normality to illness or acute illness, which evidences the capacity of the methodology to evaluate the cardiac dynamics with presence of arrhythmias, counting also its grade of evolution to determine which is more or least closest to the acute state. This is evidenced with the results of the statistical analysis where a 100% of sensibility and specification was found, as well as a Kappa coefficient of 1, pointing out that all the analyzed dynamics where correctly differentiated by means of the mathematical methodology applied.

It was evidenced, as well, that with the application of this methodology, the total of the dynamic evaluation can be accomplished, allowing to differentiate normality and the different states of abnormality, independent from isolated manifestations such as the extra-systole. In this sense, such extra-systole does not affect the mathematical diagnosis issued for each one of the evaluated dynamics.

DISCUSSION

This is the first work that performs a diagnostic application of the methodology based on the exponential law to cardiac arrhythmias, achieving a quantification of their severity. This helps differentiating arrhythmias in an objective and reproducible way and makes the methodology an important tool in their characterization, treatment and prevention, showing that isolated manifestations such as extra systoles do not affect all of the dynamics and therefore do not change the mathematical diagnosis.

The simulations performed in this study are within clinical ranges and are useful in the construction of attractors. For this reason, it is not necessary to have all the information of frequencies rigorously over time, because the actual data of the consecutive frequencies are included in the simulation and therefore do not affect the result. The application of the occupation law to the attractors with respect to their degree of irregularity used in cardiac dynamic reveals a mathematical measure adjustable to any event, regardless of age, allowing establishing a diagnosis for any adult heart function of individuals aged 21 or older.

This work showed that when evaluating a given cardiac dynamic, using the mathematical diagnosis to determine its state of normality or abnormality, isolated manifestations do not influence the dynamic. This was also shown in a previous study which analyzed arrhythmias based on probability theory (Rodríguez *et al.,* 2012c). This allows evaluating cardiac dynamics with varying degrees of extra systoles, since they do not affect the entire system. Also, it should be noted that this type of changes in the system can occur throughout the Holter tracing and in all the dynamics, thus confirming that a greater or lesser number is not considered relevant in the mathematical diagnosis since it has a measure that quantifies the overall self-organization of the dynamic regardless of isolated manifestations.

The study of the cases diagnosed as normal from the conventional parameters, but with indications

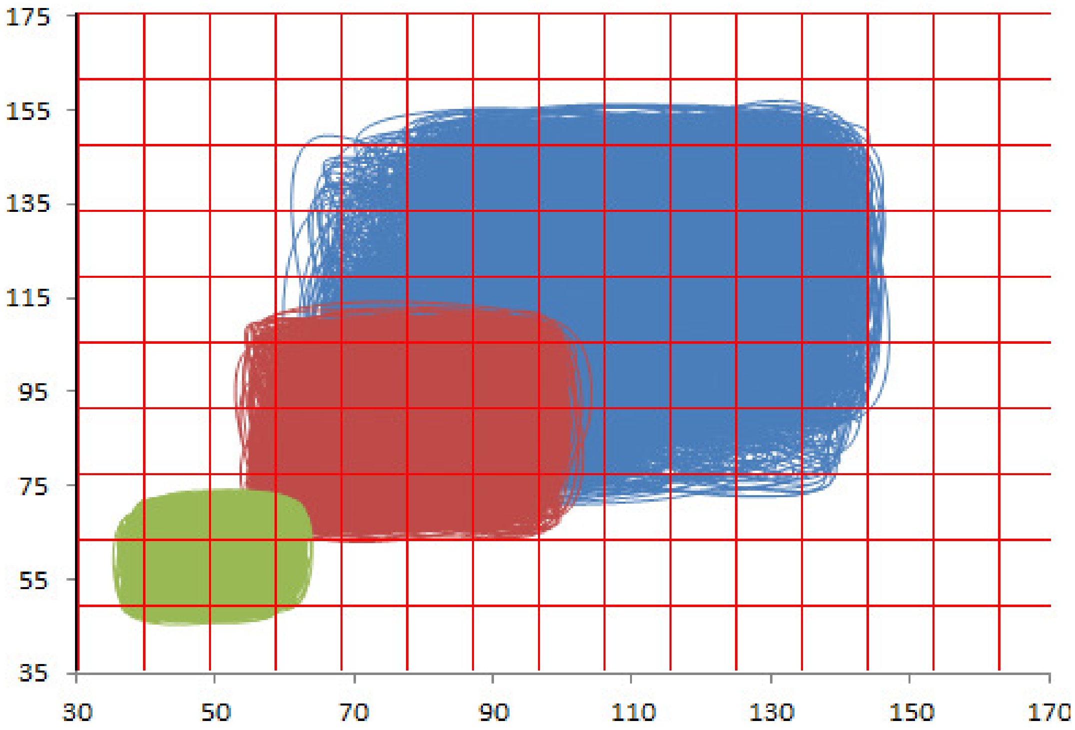

Figure 1. Attractors of the cardiac dynamic of three holters, where the x axis corresponds to a heart rate and the y axis corresponds to the next heart rate in the sequence. The blue one corresponds to a normal patient without indications, symptoms or risk factors; the red one corresponds to a holter with cardiac arrhythmia, mathematically diagnosed in evolution; the green one corresponds to a holter with AMI, with a mathematical diagnosis of acute pathology. The graph is superimposed on the Kg grid used in the box counting method

such as syncope, heart palpitations, hypertension, chest pain being studied or tachycardia being studied revealed that the methodology allows to detect slight alterations of the cardiac dynamics, not identified as pathological from the conventional parameters, so they may be an indicator of warning, pointing the cases that may need closer monitoring in time to prevent their evolution to clearly pathological states. Furthermore, it is evident that arrhythmias with mathematical values of evolution cannot be distinguished from these cases, demonstrating that this methodology shows the severity level of the dynamic independently of the specific pathology.

The figure 1 shows three different dynamics in the evolution from normality to acute disease. It is possible to see that the evolution from normality to acute disease is characterized with a progressive decrease of spatial occupation of the attractor. This decrease, which is visually observed, is quantified on the methodology developed, allowing to establish mathematically the differences between normality acute disease and evolution. Although the physical-mathematical diagnosis given by the occupancy of spaces by the attractor allows quantitatively clarifying the severity of the arrhythmia, the results of this and other studies (Rodríguez *et al.,* 2012c) indicate that the implications of this new methodology need to be refined in future research to find specific correlations related to the quantification of arrhythmias.

The applied methodology is based on a simple experiment that allows differentiating between normality and disease, the evolution between the two and the generalization for any case in the universe regardless of epidemiology (see figure 1). It is based on physics' general and causeless conception and thus replaces epidemiology, which is based on causes and population analysis. Such perspective in medical research has achieved successful results both experimentally and clinically in different areas.

Such is the case of a fractal generalization that allows deducing all possible coronary arteries in the process of restenosis (Rodríguez *et al.,* 2010d), or the development of diagnostic methods of fetal cardiac dynamics (Rodríguez, 2006a). Other publications show a Holter's diagnostic methodology of clinical application, based on the laws of probability and entropy, which allows differentiating normal, chronic, acute illness, and evolution between these states, by analyzing heart rates and entropy proportions in the geometrical attractor (Rodríguez, 2010a; Rodríguez *et al.,* 2010f).

This methodology was subsequently applied to the study of the cardiac dynamic of patients in the coronary care unit, confirming the diagnostic predictions (Rodríguez, 2011a). In fact, a case of a patient in the immediate post-operative of cardiac surgery that showed evolution of the cardiac system towards acute illness from a mathematical diagnosis, but did not show any clinical symptoms was predicted. Also, in the epidemiology field, concepts of entropy have been applied to develop a predictive methodology of malaria outbreaks every 3 weeks (Rodríguez, 2010b).

Isolated fractal dimensions do not always differentiate between normality and disease, plus, they present limitations in other research areas such as cancer (Lefebvre and Benali, 1995; Baish and Jain, 200). In contrast, Rodriguez et al. have developed methodologies that can be applied to particular cases, regardless of statistical and epidemiological methods, because they use new mathematical concepts based on fractal geometry (Rodríguez *et al.,* 2010d). Such is the case of a methodology developed to diagnose preneoplastic and neoplastic cells that allow mathematically diagnosing ASCUS cells (Rodríguez *et al.,* 2010e).

In other areas of medicine, the application of physical and mathematical theories have also allowed developing results of clinical application in immunology and molecular biology (Rodríguez, 2008; Rodríguez *et al.,* 2010c). Also, set theory applied to leucocyte and lymphocyte population revealed an objective and reproducible mathematical self-organization of clinical application to predict ranges of CD4/μl that can be used to reduce costs and resources (Rodríguez *et al.,* 2012a).

In this investigation, fractal dimension is taken as the whole and the attractor's occupation spaces are taken as the parts in order to reveal a law between these mathematical relationships, from which all the possible discrete fractal attractors can be deduced. With these fractals' occupation spaces, clinical behaviors of normality and disease can be distinguished. Also, unlike the conception of normality and disease of dynamic systems developed by Goldberger (2002), there is no randomness in this work, but the order of a law for any dynamic and the limits to study evolution between these behaviors, showing a physical and mathematical self-organization that explains this phenomenon.

This paper shows how laws and generalizations from a physics perspective can be developed for every case in the medical universe even though they are based on particular cases and a simple experiment. From a medical point of view, the early diagnosis and the quantification of the arrhythmia's severity contribute to a more rapid and effective decision regarding therapy and adds strong evidence for the referral of patients to higher levels of healthcare.

In modern physics, with statistical mechanics (Feynmann *et al.,* 1964a; Tolman, 1979), quantum mechanics (Feynmann, 1964b) and chaos theory (Devaney, 1992; Peitgen, 1992a; Cruchfield *et al.,* 1990), causality is no longer a basis for understanding nature. In this research, the law that was found suggests a causeless physical and mathematical order underlying the chaotic cardiac dynamic. This law is therefore predictive and applicable to any other chaotic dynamic, suggesting a law for any chaotic process that can be clinically reproduced regardless of initial conditions and avoiding the problems of chaos theory unpredictability. It is possible, based on deterministic chaotic dynamics of physiological phenomena (Goldberger *et al.,* 1990; Goldberger *et al.,* 1996; West, 1990) to generalize physiology based on physics laws, justifying physiology from physics.

These mathematical limits allow designing programming for pacemaker and would be very important for physical and mathematical evaluation of pharmacological efficacy. It would be important to compare this method with the conventional one in mortality studies and in studies where normality is compared with disease in order to confirm its clinical applicability.

Dedication

To our children

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