International Research Journal of Engineering Science, Technology and Innovation (IRJESTI) (ISSN-2315-5663) Vol. 2(3) pp. 51-56, March, 2013
Available online http://www.interesjournals.org/IRJESTI

Full Length Research Paper

Stress and strain paths in deep drawing of cylindrical cup

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Accepted February 26, 2013

Stress-strain relationship at large plastic deformation during sheet metal forming process are accompanying with numerous problems. These problems include the predicting important parameters which effect accuracy and durability of the products made by different methods. Most of presented methods for analysis of sheet metal forming do not taking into account deformation hardening and thickness changes during the forming process and are limited in application, especially in the accuracy of final solutions. Therefore it important to find a mathematical model which can correctly analysis forming problems of sheet metal by taking in to account the thickness changes and deformation hardening. At axi-symmetric forming of sheet metal the accuracy of products depends on the stability of dimension which for an isotropic material in the absence of residual stress is determined by thickness distribution on forming products. Durability of products under the same conditions is defined by total distribution of yield stress of the material. In forming processes of sheet metal the yield stress is a function of effective stress which defined by main deformations parameters. Hence, the problem of predicting of durability and accuracy parameters is reduced to the determination of value and distribution of equivalent deformation in the main direction. This research present a new method to analysis the axi-symmetric forming condition of sheet metal in the conditions of thickness change and deformation hardening and this method is applied for analysis deformation of thin ring plates.

Keywords: Stress path, strain path, deep drawing, cylindrical cup, sheet metal forming.

INTRODUCTION

In the deep drawing process a sheet, the ‘blank’, is clamped between a die and a blank holder. The specific shape of the punch and die is transferred to the sheet during the forming operation. A principle outline of this process is given in Figure 1. In practice, the material flow during deep drawing can hardly be controlled by the blank holder. To improve the material flow control, draw beads can be used which are flow linesappearing on the die surface. Because of these draw beads the material flow is more restrained causing a change of the strain distribution with consequently thinning of the sheet (Marciniak et al., 2002; Popov, 1977). Figure 2 shows deep drawing sequence and final cup.

Consider a blank with initial diameter D0 and central hole diameter d0. According to D0 ⁄ d0 ratio, it is possible to find 4 different conditions which are represent in Figure 3 (Popov, 1977). Sheet metal forming process is accompanying by deformation hardening process and thickness changes

in drawn cylindrical cup. But most of researches in modeling of this process, do not take in to account these parameters, so their process analysis are not in comparable with real process. To achieve a correct analysis, it is important to find the effects of these parameters on forming conditions (Hill, 1998).

Analysis of thin ring plates deformation

Consider thin ring plate, Figure 4a, having the sizes

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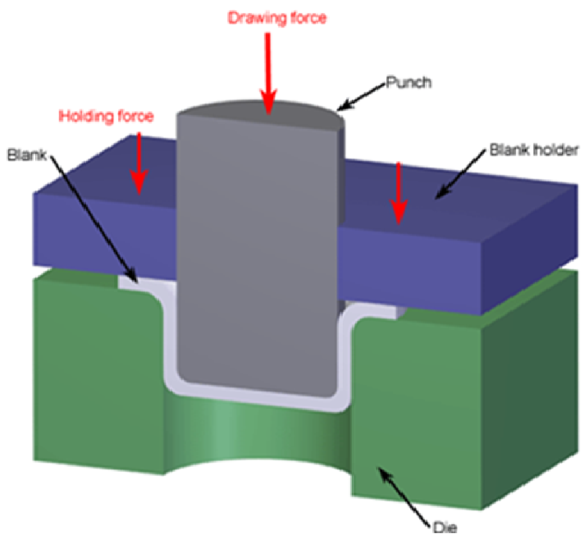


Figure 1. Deep drawing scheme.

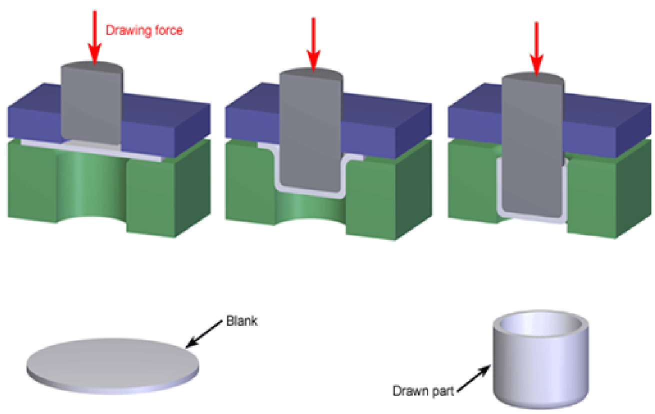


Figure 2. Deep Drawing sequence.

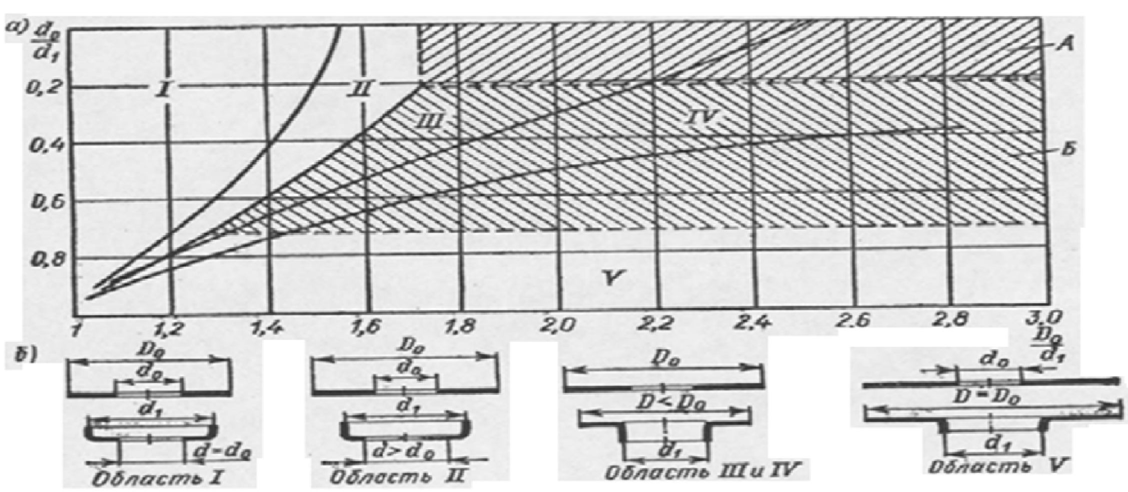


Figure 3. Deep Drawing and Flanging process at different conditions
I- Deep drawing with changing in central hole diameter
II- Deep drawing without changing in central hole diameter
III-Flanging without changing in central hole diameter
IV-Flanging with changing in central hole diameter

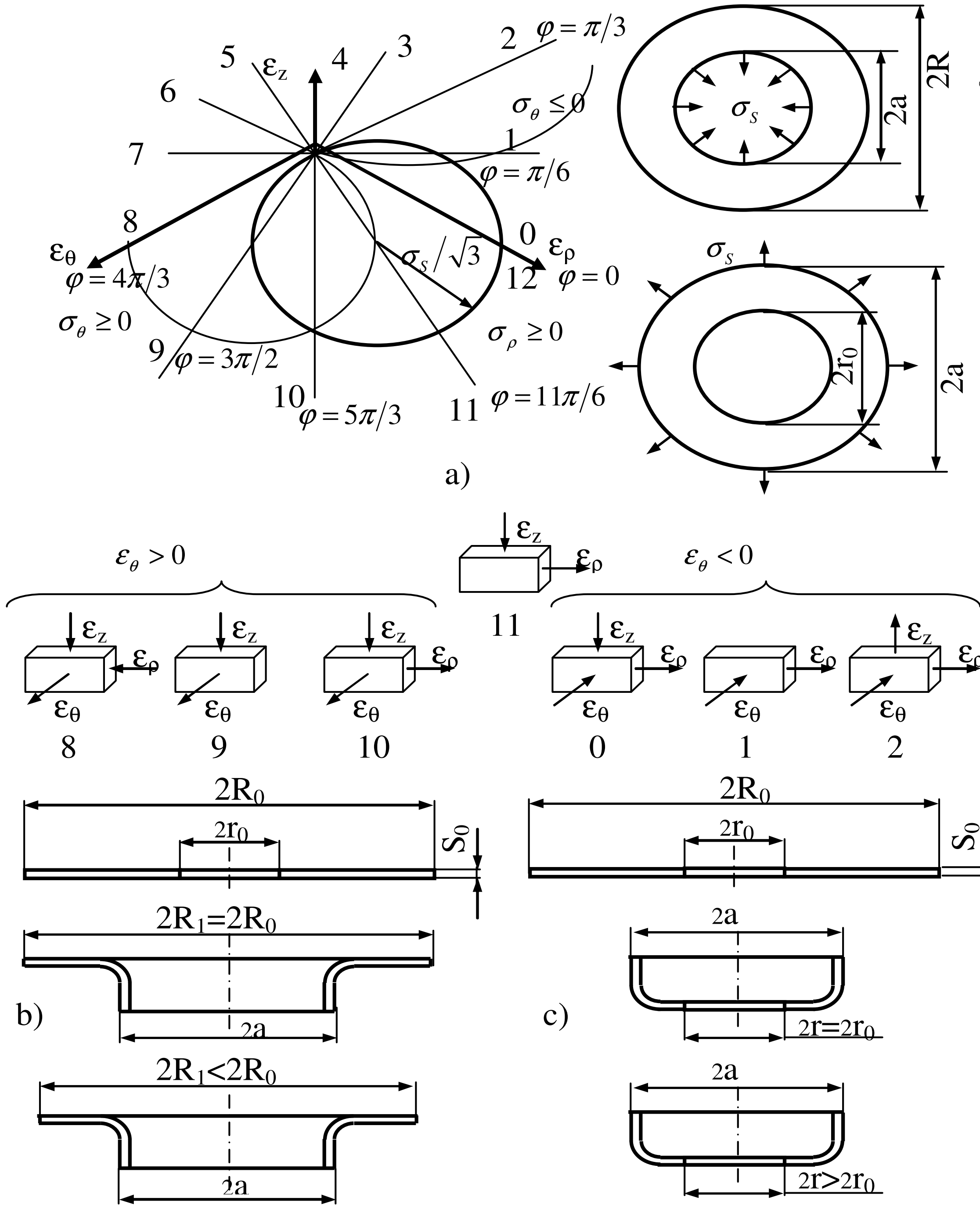


Figure 4. Stress condition on π-plane (a) and deformation types (b-flanging, c-drawing) (Nazaryan et al., 2009).

R_0, r_0, s_0. At certain dimensional characteristics of the plate and forming tool the following variants of forming are possible:

- Flanging of central hole at constant or variable value of diameter of an external contour (Figure 4 b).
- drawing of cylindrical cup at constant or variable value of diameter of the central whole (Figure 4c).

The given character of forming allows us to conventionally divide deformation of ring plate into two components: stretching of ring plates with the sizes R_0, a, s_0 and a, r_0, s_0 under the yield stress σ_s accordingly applied to internal and to external contours. It is obvious that at such condition the stresses arising due to material bending on radial flanges of deforming tools are not taken into consideration, and the size of the ring dividing the drawing zone from flanging zone is replaced with conventional circle with radius a.

Taking into account deformation equations, it is possible to show stress-strain relation in a simple equation as follow (Arab and Nazaryan, 2009; Nazaryan and Arab, 2009; Nazaryan et al., 2009).

$$d\sigma_\rho = \sigma_s d\varepsilon_i \qquad (1)$$

Integration of this equation is difficult because increment of equivalent strain does not equal to increment intensity of main strain. It is easy to show that such equality is possible only at proportional change of main strains that corresponds to radial ways of deformations (Arab et al., 2009; Najmeddin Arab, 2011).

At such an assumption it follows from (1):

$$\sigma_\rho = \frac{A}{n+1}\varepsilon_i^{n+1} + c \qquad (2)$$

Integration constant is found from boundary conditions on which for contours free from load radial stresses equal zero. Taking into account boundary conditions and by equating the integration result to equation (3), we will have

$$\varepsilon_i^{n+1} - \varepsilon_{kp}^{n+1} = (1+n)\varepsilon_i^n \frac{2}{\sqrt{3}}\cos(\varphi + \pi/6) \qquad (3)$$

Which ε_{kp} is the equivalent strain of the edge element.

The received solution allows establishing interrelation between coordinates of the considered material parameters and the angle of deformation state kind. For this purpose, by differentiating at ε_{kp} =0.

$$\frac{d\rho}{\rho} = (1+n)\cdot\left(\sin\varphi\cos\varphi + \frac{\sqrt{3}}{2}\sin^2\varphi + \frac{\sqrt{3}}{6}\cos^2\varphi\right)d\varphi \qquad (4)$$

$$\ln\rho = (1+n)\cdot\left(\frac{\sqrt{3}}{3} - \frac{1}{4}\cos 2\varphi - \frac{\sqrt{3}}{12}\sin 2\varphi\right) + C \qquad (5)$$

Integration constant in (5) is obtained from the following boundary conditions: for external ring plate $\rho = R_0,\ \varphi = \pi/3$; for internal ring plate $\rho = r_0,\ \varphi = 4\pi/3$. Taking into account boundary conditions expression (5) becomes

$$\frac{\rho}{R_0} = \exp\left[(1+n)\cdot\left(\frac{\sqrt{3}}{3}(\varphi - \pi/3) - \frac{1}{4}\cos 2\varphi - \frac{\sqrt{3}}{12}\sin 2\varphi\right)\right] \qquad (6)$$

for internal ring plate

$$\frac{\rho}{r_0} = \exp\left[(1+n)\cdot\left(\frac{\sqrt{3}}{3}(\varphi - 4\pi/3) - \frac{1}{4}\cos 2\varphi - \frac{\sqrt{3}}{12}\sin 2\varphi\right)\right] \qquad (7)$$

Substituting in (6) and (7) the limiting values of angles of deformation state kind $\varphi = 0,\ \varphi = \frac{5}{3}\pi$ at which stretching stresses reach the value equal to material yield point, we will receive the relation of the greatest sizes of ring plates (Nazaryan and Konstantinov, 1999).

$$\frac{a}{r_0} = \frac{R_0}{a} = \exp\left[(1+n)\cdot\left(\frac{1}{4} + \frac{\sqrt{3}\pi}{9}\right)\right] \approx \exp[0{,}854\cdot(1+n)] \qquad (8)$$

At $n = 0$ limiting values of relations R_0/a and a/r_0 (Limit Drawing Ration (LDR)) are equal to 2.35 which coincides with the result of works (Arab and Nazaryan, 2009; Nazaryan and Arab, 2009; Nazaryan et al., 2009; Arab et al., 2009; Najmeddin, 2011) and is close enough to the values usually seen in experiments (Marciniak et al., 2002; Popov, 1977; Hill, 1998). Using value of equivalent strain, it is easy to define the main strains for the initial plastic state of ring plates:

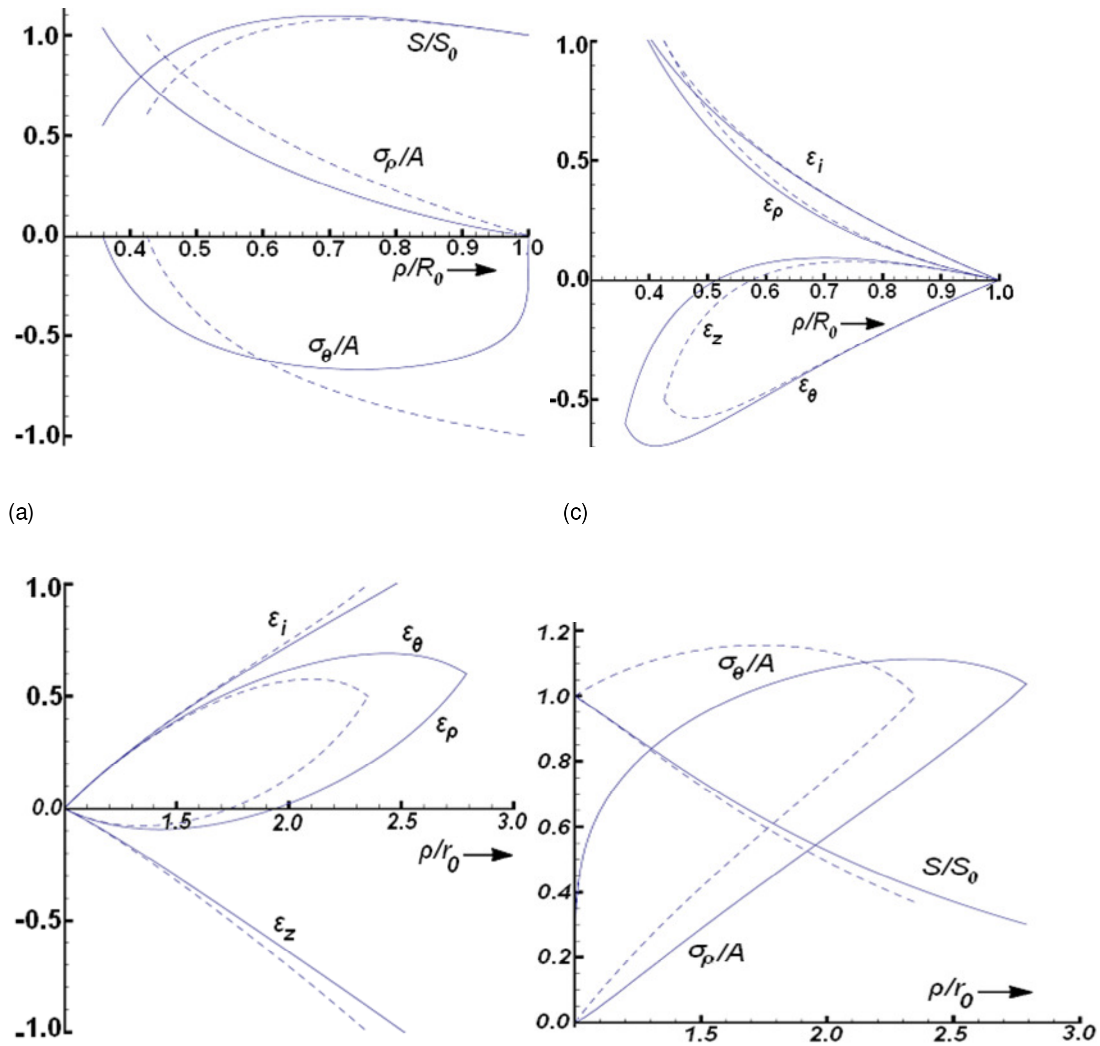
$$\varepsilon_\rho = \frac{1+n}{2}(1 + \cos 2\varphi - \frac{\sqrt{3}}{3}\sin 2\varphi)$$
$$\varepsilon_\theta = -\frac{1+n}{2}(\cos 2\varphi + \frac{\sqrt{3}}{3}\sin 2\varphi) \qquad (9)$$
$$\varepsilon_z = -\frac{1+n}{2}(1 - \frac{2\sqrt{3}}{3}\sin 2\varphi)$$

Thus, dependences (1), (3), (6), (7) and (9), being the parametrical solution of the problem, completely define the stress-strain state of ring plates taking into account the interconnected change of material thickness and deformation hardening.

It is obvious that by eliminating parameter φ it is possible to receive the dependences characterizing the distribution of main stresses and strains on coordinate of material parameters for the initial plastic states of plates.

RESULTS AND DISCUSSION

On Figure 5 a, b the distribution graphs of equivalent and main strains are presented, and on Figure 5 c, d the distribution graphs of main stresses and thickness for internal and external ring plates, are accordingly presented.



(b) (d)

Figure 5. The distribution of main stresses and thickness (a,b) and equivalent main strains for external and internal ring plates (c,d) of an equivalent and main strains for external and internal ring plates (- - - - n=0 , _____ n=0.2).

From graphs some characteristic features of forming ring plates follow. For external ring plate equivalent strain and radial component differ slightly. On the boundary area the external plate experiences thickening deformation, and absolute value of circular deformation at some distance from the internal contour reaches its maximum.

For internal ring plate equivalent strain and deformation by thickness slightly differ in absolute value. At internal contour the internal plate undergoes deformation of radial compression, and circular deformation at some distance from external contour reaches its maximum. Deformation hardening, not changing the deformed state in quality, essentially influences the stress state. Coordinates of material parameters, which limit characteristic kinds of deformation, may be defined from equations (8), (9).

The determined distribution of strains may be used both for definition of the initial sizes of ring plates on the set sizes of the blank and for optimization of parameters of the forming process.

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