

Full Length Research Paper

Modeling and optimal control algorithm design for HVAC systems in energy efficient buildings

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This report focuses on modeling the thermal behavior of buildings and designing an optimal control algorithm for their HVAC systems. The problem of developing a good model to capture the heat storage and heat transmission properties of building thermal elements such as rooms and walls is addressed by using the lumped capacitance method. The equations governing the system dynamics are derived using the thermal circuit approach, and by defining equivalent thermal masses, thermal resistors and thermal capacitors. In the control design part, we have introduced a new hierarchical control algorithm which is composed of lower level PID controllers and a higher level LQR controller. The optimal tracking problem is solved in the higher level controller where the interconnection of all the rooms and the walls are taken into consideration. The LQR controller minimizes a quadratic cost function which has two quadratic terms. One takes into account the comfort level and the other represents the control effort, i.e. the energy consumed to operate the HVAC system. There are two tuning parameters as the weight matrices for each of these two terms by which the performance of the controller can be tuned in different operating conditions. Simulation results show how much energy can be saved using this algorithm.

Keywords: Modeling, control, tracking, simulation.

INTRODUCTION

The buildings in India consume more energy than any other sector of the India economy, including transportation and industry, says the India government. Buildings account for approximately 40% of world energy use, thus contributing 21% of greenhouse gas emissions. In the India alone, buildings contribute 1 billion metric tons of greenhouse gas emissions. With growing environmental awareness and uncertainty in global energy markets, energy efficient buildings hold great appeal for consumers, corporations, and government agencies alike. According to the

Indian. Energy Information Agency, homes and commercial buildings use 71% of the electricity in the India and this number will rise to 75% by 2025. Homes account for 37% of all India electricity consumption and 22% of all India primary energy consumption (EIA 2005). This makes home energy reduction an important part of any plan to reduce India contribution to global climate

change.

Motivation

HVAC the main energy consumer in Buildings in 2001 building heating ventilation and air conditioning (HVAC) systems accounted for approximately 30% of total energy consumption in the India. This is greater than transportation, which accounted for approximately 28% of total energy consumption. However, the energy consumed by HVAC systems is less evident and distributed across residential, commercial and industrial sectors. HVAC systems, in particular cooling, are one of the fastest growing energy consumers in the India This trend started in the 1970s, and continues today. However, much of this growth has been offset by gains in efficiency. There is still much room for improvement in the efficiency of such systems with technology that

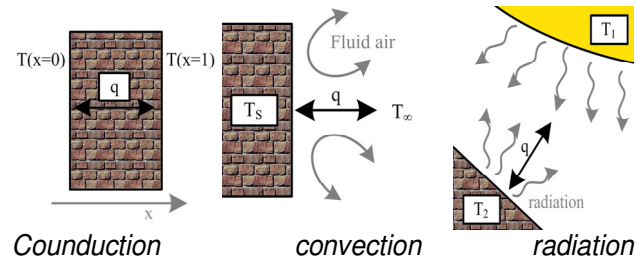


Figure 1. Mechanisms of heat transfer

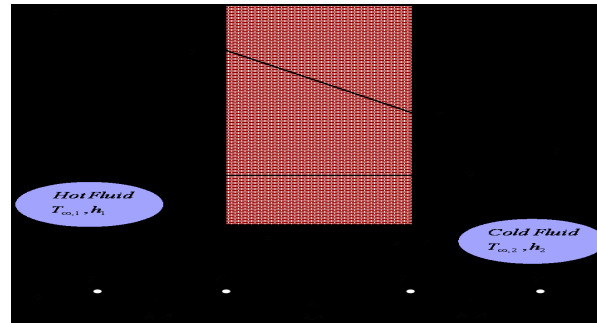


Figure 2. Heat transfer through a plane wall. Temperature distribution and equivalent thermal circuit

already exists

Objective of this project

In this project we address the problem of designing a new control algorithm for HVAC systems that improves the comfort level of the occupants in buildings and at the same time consumes less energy to reach this goal.

The control algorithm is a hierarchical control consisting of two levels of controllers; higher level and lower level controllers. This report presents an optimal control algorithm that takes into account the time varying behavior of thermal loads and operates more efficiently and more economically while keeping the desired comfort level.

Heat Storage

A basic property of materials is specific heat capacity c_p , which is the measure of heat or thermal energy required to increase the temperature of a unit quantity of a substance by one unit. More heat is required to increase the temperature of a substance with high specific heat capacity than one with low specific heat capacity. For an object with mass m and specific heat capacity, a rate of change of temperature T corresponds to the heat flow, denoted by Q ; In the more familiar parlance of electrical engineering,

M_{cp} is capacitance,
 T is the rate of change of potential and
 Q is current.
 $Q = m_{cp} \cdot T$

Heat Transfer

Heat transfer takes place via the mechanisms of conduction, convection, and radiation as shown in Figure (1).

Thermal resistance

In particular there exists an analogy between the diffusion of heat and electrical charge. Just as an electrical resistance is associated with the conduction of electricity, a thermal resistance may be associated with the conduction of heat (Figure 2)

$$R_{t; \text{cond}} = \frac{T_{s;1} - T_{s;2}}{q_x} = \frac{L}{kA}$$

Similarly for electrical conduction in the same system, Ohm's law provides

an electrical resistance of the form

$$R_e = \frac{E_{s;1} - E_{s;2}}{I} = \frac{L}{\sigma A}$$

Thermal potential

As it was discussed above, in steady state conditions we

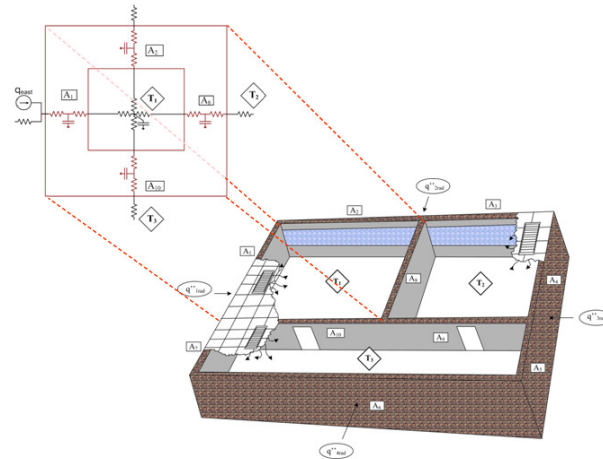


Figure 3. Simple three-room building with heat transfer through exterior and interior walls.

can define thermal resistances for different heat transfer modes such as conduction and convection. Accordingly, we can construct an equivalent thermal circuit to analyze the thermal behavior of the system. It was also shown that the equations derived here are analogous to the corresponding equations in an electrical circuit.

The other similarity that is noticed is the notion of thermal potential or temperature in thermal circuits which is analogous to the concept of electrical potential in electrical circuits. The temperature (thermal potential) of a point is fixed in steady state heat transfer, while it varies with time in transient heat transfer or heat storage.

Thermal capacitance

In order to analyze the transient thermal behavior of the building model, we need to introduce the concept of thermal capacitance. During transient heat transfer the internal energy (and accordingly temperature) of the materials change with time. Thermal capacitance or heat capacity is the capacity of a body to store heat. It is typically measured in units of $(J = C)$ or $(J = K)$

If the body consists of a homogeneous material with sufficiently known physical properties, the thermal mass is simply the mass of material times the specific heat capacity of that material. For bodies made of many materials, the sum of heat capacities for their pure components may be used in the calculation. In the context of building design, thermal mass provides inertia against temperature fluctuations, sometimes known as the thermal wheel effect. For example, when outside temperatures are fluctuating throughout the day, a large thermal mass within the insulated portion of a house can serve to "attend out" the daily temperature fluctuations, since the thermal mass will absorb heat when the surroundings are hotter than the mass, and give heat

back when the surroundings are cooler. This is distinct from a material's values, which reduces a building's thermal conductivity, allowing it to be heated or cooled relatively separate from the outside, or even just retain the occupants' body heat longer. In order to capture the evolution of temperature of walls and rooms we assign a capacitance with capacity $C = mcp$ to each node in the thermal circuit. Notice that bodies of distributed mass like walls and air are considered as nodes in our modeling.

Plant Modelling

We are now ready to derive the governing heat transfer equations for the temperature distribution in walls and rooms of a simple building. The heat transfer and storage equations compose a simple plant model representing a three room building. A more accurate model of temperature is significantly more complex and it does not facilitate the derivation of control laws. We assume that the specific heat of air, c_p , is constant at 1.007. In reality, c_p is 1.006 at 250 K and 1.007 at 300 K, so our assumption is accurate to within 0.1% error over the range of temperatures that would occur in a building. All rooms are at the same pressure used in the heating and cooling ducts. Air exchange between a room and vent is then isobaric, so the air mass in the room will not change in the process. We denote the air mass in the room by m and the rate of air mass entering the room, and also leaving the room, by radioactive heating for each building face (N; S; E; W) is an input to the plant model. In a real building, the changing position of the sun through the day, and variations in atmospheric attenuation here due to lack of exact data for the intensity of irradiation from the sun for a given time in a day, we use a sinusoidal input for the sun irradiation. We ignore radioactive coupling between inner building walls; as the temperature difference between pairs of walls should be small, the

effects of interior radioactive coupling are likely to be minimal. For a single room, the thermal model that results from our simplifying assumptions is presented as Figure (3). Also the detailed view of room number 1, coupled to its four surrounding walls, is given in detail. The temperature of room 1 is called T_1 while the temperature of the adjacent rooms 2 and 3 are called T_2 and T_3 respectively. The thermal capacity or thermal mass of room i is denoted by C_{ri} which is equal to the mass of the air in room i , m_i times the speci_c heat capacity of air, c_p , i.e.

$$C_{ri} = m_i c_p$$

Controller Design

In order to investigate how new control techniques can help improve energy efficiency of large buildings, a scalable thermal model for rooms and buildings was developed in Section 3 and 4. Scalability is important when analyzing the heat transfer behavior of large buildings. Thus we tried to keep the state space model representation of the system as general and standard as possible so that for example a model for a 3-room building can be easily extended to a model for a 30-room building. In this section, we introduce the classical controllers for HVAC systems and also the modern optimal controllers. Although the model derived in the previous section is in continuous domain, here we discuss the control problem in the discrete domain. Usually when the plant model is in continuous domain, there are two possible approaches to design and implement the controller. The first approach is to use a continuous plant model and design a continuous controller but implement it digitally. The second approach is when we use a discrete plant model and design a discrete controller and implement it digitally. Each method has its own advantages and disadvantages, which depends on the time constants and the sampling time. For this project we have chosen to use the second method, i.e. discretizing the plant model and designing a discrete controller, and then implement it digitally.

Classical HVAC control techniques

Classical controllers for HVAC systems include on-off controller and Proportional-Integrator-Derivative (PID) controllers. These controllers have a simple structure and low initial cost. However in long term these controllers are expensive due to their low energy efficiency. on-off controllers work either in the "on" or "off" state providing only two outputs, maximum (on) and zero (off). The limited functionality of on-off controller makes it inaccurate and not of high quality. PID controllers which have advantages such as disturbance rejection and zero

steady state offset have been commonly used in many HVAC applications. The main drawback of classical air conditioning control systems is that most HVAC systems are set to operate at design thermal load, while actual thermal loads are time varying and depend on the environmental factors like outside weather conditions, and the number of people in the building.

Hierarchical control algorithm

In any control algorithm for HVAC systems, sensing and actuation are managed locally at the room-level. To achieve building-level energy-optimality, the rooms cannot act autonomously. To minimize building-level energy consumption, the actions relative to the rooms must be coordinated. In this report, the coordination between the rooms is achieved by using hierarchical control. We introduce two levels of control over the system, consisting of PID as lower level and an LQR as higher level controller. Typically the controllers used for HVAC systems are PID controllers. Lower-level (PID) control governs sensing and actuation within a single room. The higher-level (LQR) control is supposed to determine the optimal input to the system so that the cost function which is a combination of deviation from set point temperature set by the user and the control effort can achieve its minimum possible value. By applying the optimal input, cooling/heating air flow to the rooms, we still remain in the comfort zone defined according to the psychrometrics charts. The difference of the proposed control algorithm in this work with the classical control techniques is that the desired temperature for every thermal zone is not directly fed into a local controller but into a higher level controller that has a global view of the current and desired state. The higher level controller (LQR) determines the appropriate set points for the lower-level controllers of each room in a building. Higher-level and lower-level controllers can be referred to as room-level and building-level controllers respectively.

Room level PID control

As mentioned above, the lower-level control is accomplished using a PID controller. which x represents the state and represents the inputs, which are the temperatures of the walls and rooms and the air flow mass into the rooms, respectively. Instead of allowing the set point to be controlled by a thermostat, the user set point and state of the room are sent to the higher level controller i.e. a linear-quadratic regulator which optimally calculates the set point for the lower-level controller and sends it back to the lower level PIDs. Therefore all the rooms are controlled locally by PID controllers which track the set point given by the higher level LQR

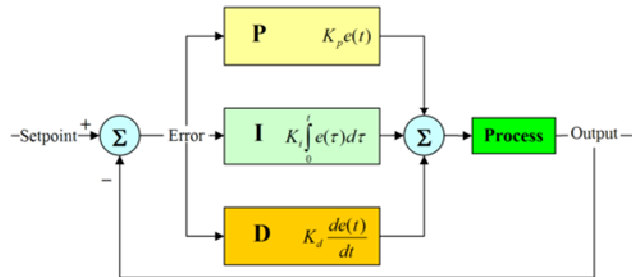


Figure 4. Typical PID controller

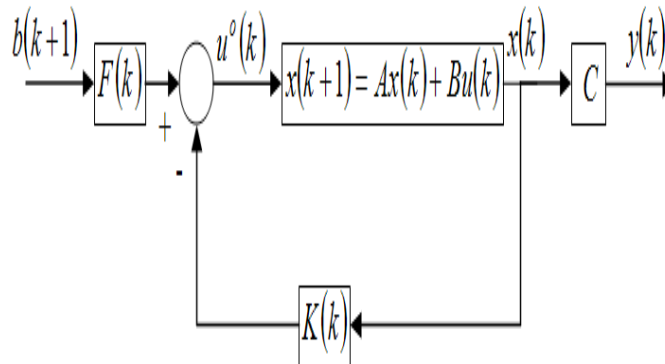


Figure 5. Block Diagram for the derived Optimal Control

controller. The task of the LQR controller is to feed the optimal set point to the PID controllers. (Figure 4)

Building-Level Linear Quadratic Regulator

In optimal control, one attempts to use a controller that provides the best possible performance with respect to some given measure of performance. For instance, we defined the controller that uses the least amount of control-signal effort to take the output to zero. In this case the measure of performance (also called the optimality criterion) is the control-signal effort. In general, optimality with respect to some criterion is not the only desirable property for a controller. One would also like stability of the closed-loop system, good gain and phase margins, robustness with respect to unmodeled dynamics, etc. In this section we review the concept of Linear Quadratic Regulator (LQR) controllers that are optimal with respect to energy-like criteria. These are particularly interesting because the minimization procedure automatically produces controllers that are stable and somewhat robust. In fact, the controllers obtained with this procedure are generally so good that we often use them even when we do not necessarily care about optimizing for energy. Moreover, this procedure is applicable to multiple-input/multiple output (MIMO) processes for which classical designs are difficult to apply. All mentioned above are the reasons why we are using LQR as the

higher level controller. We should also say that this higher level control can be implemented using other control techniques such as model predictive control

Controllability and observability

The controllability check shows that the system is not fully controllable (i.e. the controllability matrix is not full rank), but if we analyze the stability of the uncontrollable modes, Similarly, the observability check shows that the system is not fully observable, but the stability analysis of the unobservable modes, shows that the unobservable modes are stable, hence the system is detectable.

Optimal tracking problem

To implement the LQR controller on our plant, we need to modify the controller so that it can track a desired set point. The general form of LQR is designed to take the states of the system to zero. However we need the output of the system (i.e. the temperature of the rooms) to track the desired temperature trajectories that are set by the occupants. So we need to manipulate the general LQR formulation so that it can take the output of the to the desired output. Here we derive the Optimal Tracking Problem using LQR technique. The LQ tracking problem is formulated as follows: $\min U_0 \int J_g$ (Figure 5)

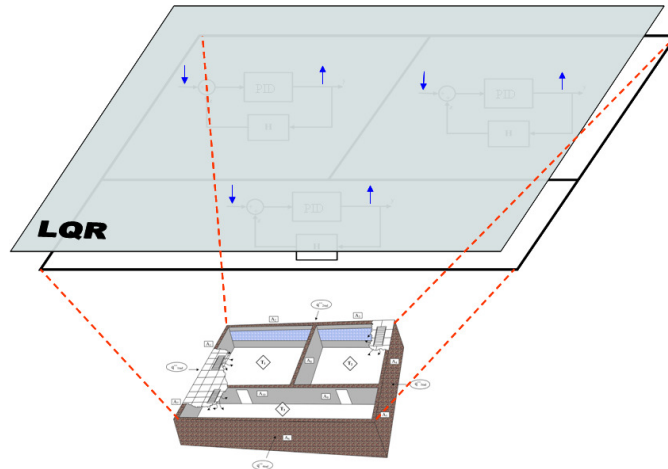


Figure 6. Hierarchical Control Algorithm

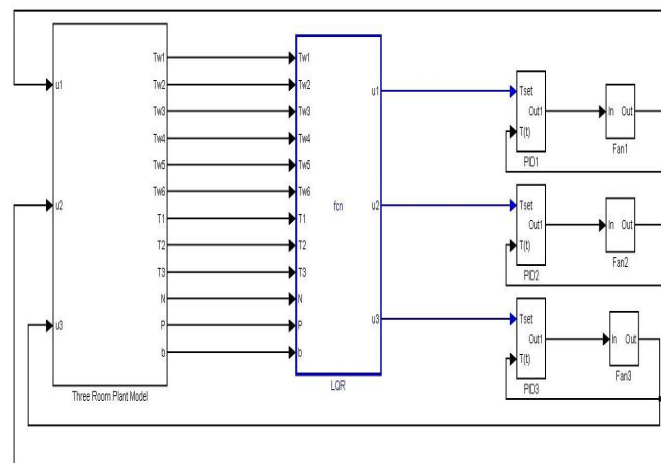


Figure 7. Interconnection of the Plant model, the lower level, and higher level controllers

Control-Algorithm Implementation

Here we will discuss more in detail the structure of the proposed control algorithm, and the implementation of the algorithm on the model. In summary, we have introduced a hierarchical control that consists of two layers of controllers. For the lower level (room level) we use PID controllers and for the higher level (building level), LQR controller is used. The LQR also needs the current temperature of the rooms. These temperatures are sensed by the temperature sensors which are mounted in specific locations in the building and are fed back to the LQR. The computations are done in the higher level controller (LQR) in order to calculate the optimal input. The input to the model is in fact the air mass flow that should enter each room through the ducts. These inputs are given to the lower level PIDs as the set points for air mass flow in each local lower level controller. The output of the PID which is a controlling

signal is given to the fans to adjust the angle of each damper in order to control the amount of air which is blown into the room. Thus the output of the fan which is optimal air mass flow is given to the plant (room). The control is now closed by sensing the current temperature of the room and feeding it back to the higher level controller (LQR). (Figure 6)

Including lower level PIDs and higher level Modelling of the heat transfer system based on the equations derived and also the implementation of the control algorithm introduced are done in Simulink. A library was also developed for future use which has some elements like the model of a wall and a room, which can be combined to make an arbitrary building. We show the interconnection of two layers of controllers which was described above. the system dynamics is solved in the left box labeled as "Three Room Plant Model" with the inputs of the block be in the mass air ow inputs from the PIDs. This block simulates the dynamic behavior of the

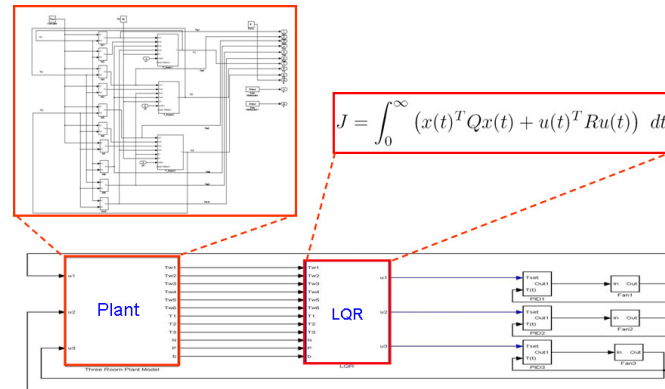


Figure 8. A detailed view of the inside of Plant and LQR blocks

model and solves for the temperatures of the rooms. These temperatures are fed to the block in the middle labeled 'LQR'. In this block the optimal tracking problem is solved with Q and R matrices, as the weights for the output and the input terms defined in the quadratic cost function which is defined in the 'LQR' block. The solution of the optimal tracking problem is the optimal input which is fed to the lower level PID controller the dynamics of the fan is considered in the block between the PID controllers and the Plant. The optimal input is fed to the PID controllers and the major task of the PID is to track this reference signal. The output of the PID is the controlling signal which is given to the fans to produce the required amount of air mass flow into the rooms. So, the loop is closed by feeding the input to the plant model. A detailed view of what takes place in the Plant block and the LQR block is shown in Figure 7 and 8

Simulation

Having the model of the building ready in Simulink, now we can implement different controller strategies on the plant and compare the responses of the system, the comfort level of the occupants, and also the energy usage in each case. The final goal of the control design part of the project is to design the best controller which is able to keep the temperature of the rooms as close to the set point temperature for each room as possible while consuming the least amount of energy. The set point temperatures are set by the building occupants. We define the concept of comfort level to be the closeness of the current temperature of the room to the temperature which is set for each room by the occupants. When the gap between the set point temperature for each room and the current temperature of that room is small we say the comfort level is higher than when this gap is larger. The other factor that we consider to evaluate the performance of a controller is energy usage. We want to have a specified level of comfort by using the least amount of energy possible. It is obvious that if we use more energy

we can raise the level of comfort by more closely tracking the set point temperature of each room. In order to make a balance between the two mentioned factors i.e. comfort level and energy usage, we have two tuning parameters. The Q and R matrices are the two parameters by which we can tune the performance of the LQR controller. Q is the weight matrix for the outputs and R is the weight matrix for the inputs in the cost function. It means that if we want to put more constraint (tighten the constraint) on the output in the sense that the output tracks the desired output more closely, we can do it by increasing matrix Q, and if we want to loosen the constraint on the output, we can do it by decreasing matrix Q. Similarly, we can manipulate matrix R in order to tune the performance of the LQR controller. This can be done by increasing and decreasing matrix R when we want to tighten or loosen the constraints on the input, respectively. Note that loosening the constraint on the input gives the input more freedom to increase, and accordingly the desired output can be tracked more closely and vice versa. The way we are going to take advantage of this property of the LQR controller is that we can play with these two parameters to tune the controller. For example, when we know that there is going to be a conference in one room of a large building, and a crowd of people will be present in the room in a few hours, we can decrease the corresponding entry of that room in matrix R. Another example would be the case when it is very important for us that the temperature of one specific room be very close to the set point value for the temperature in that room. In this case we can increase the corresponding entry of that room in matrix Q. The other example which is very common is when a room is going to be unoccupied for a known period of time. In that case we set the corresponding entry of that room in matrix Q equal to zero".

RESULTS

Simulations were done for two different cases. In the first case we only simulated the local PID controllers. The

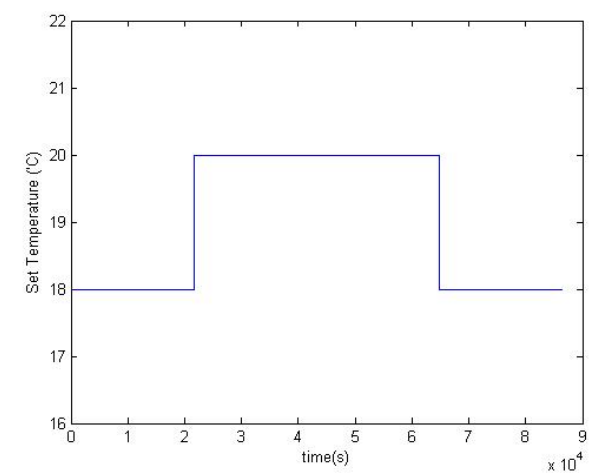
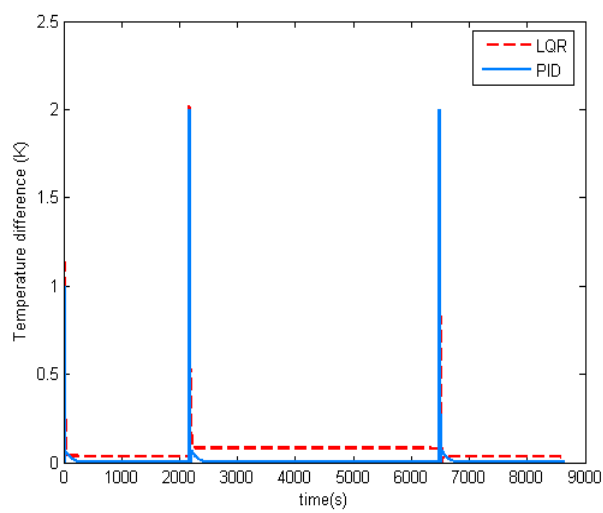


Figure 9. Temperature setpoint for the rooms



Case 1

Figure 10. Comfort Plot for case 1

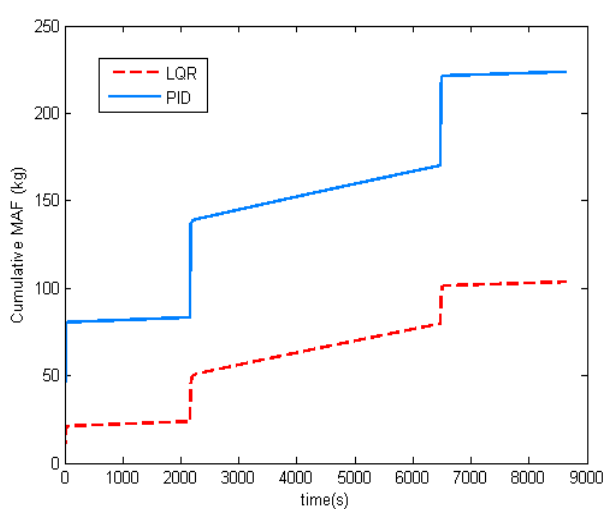
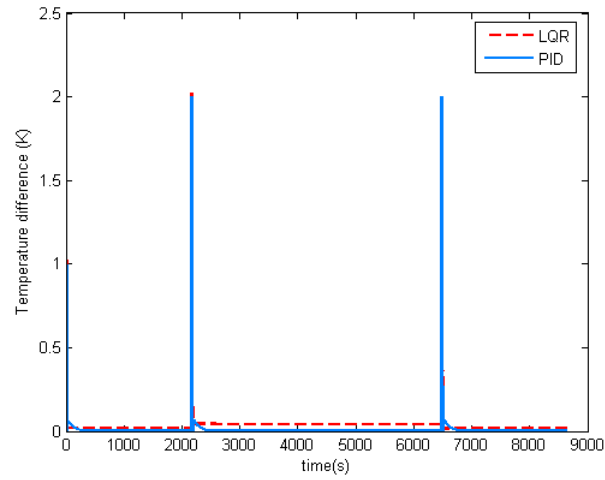
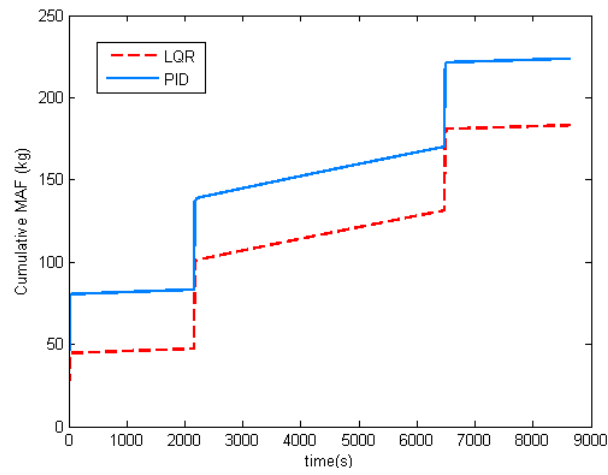


Figure 11. Energy Plot for case 1



Case 2

Figure 12. Comfort Plot for case 2



Case 3

Figure 13. Energy Plot for case 2

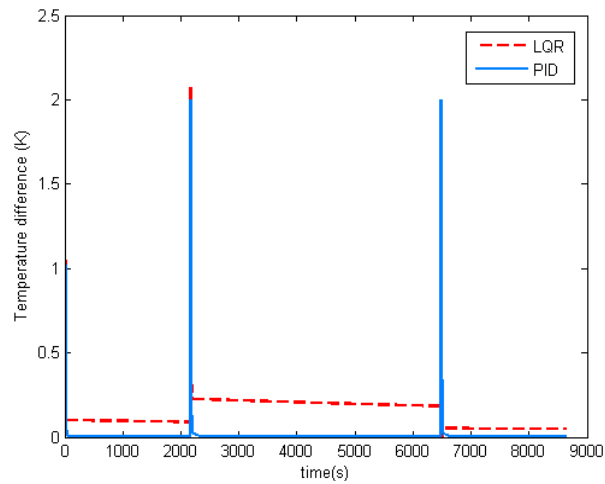


Figure 15. Energy Plot for case 3

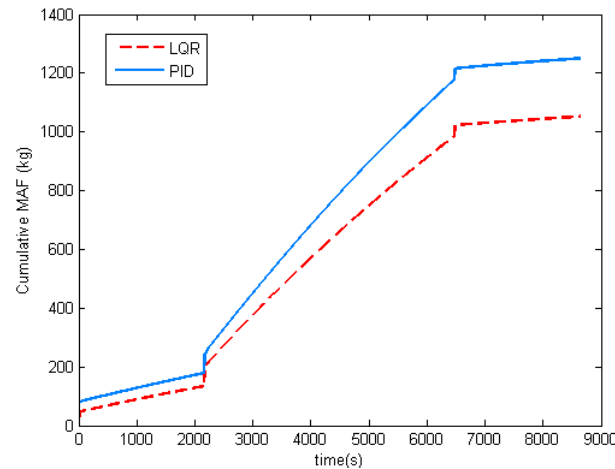


Figure 15. Energy Plot for case 3

temperature of the room is sensed and fed back to the PID controller. The PID controller just tries to track the given set point without having any idea of what the temperature trajectory is going to be like in the future. Thus in this model the input is given to the plant without any optimization process done in order to take into account the level of comfort for the occupants and also the energy which is used to reach the set points. Obviously the level of energy consumption will be higher than the case where the inputs are calculated in an optimal fashion. In the second case we have applied both the PID controller and the LQR controller to optimally track the set point temperatures of the rooms. As discussed earlier in this case, the optimal tracking problem is solved back-wards in time using dynamic programming. In this case we have two tuning parameters which can be varied to tune the performance of the controller indifferent situations. (Figure 9-15)

Verification

In this section we are using Simscape from Mathworks TM and the network node model approximation to model walls, rooms and buildings. The system allows a greater number of rooms or walls to be modeled without significant effort. Additionally, the Simscape model was verified using the analytical partial differential equations. The building model is entirely represented by electric elements using the libraries provided by Simscape. The system could be easier to scale, since there is no need to write analytical expressions.

SUMMARY AND CONCLUSION

In this report we presented a methodology to model the thermal behavior of buildings and an optimal control

algorithm for their HVAC systems. The problem of developing a thermal model to capture the heat storage and heat transmission behavior of building thermal elements such as rooms and walls was addressed by using the lumped capacitance method. The equations governing the system dynamics were derived using the thermal circuit approach, and by defining equivalent thermal masses, thermal resistors and thermal capacitors. In the control design part, we introduced a new hierarchical control algorithm which is composed of lower level PID controllers and a higher level LQR controller. The optimal tracking problem is solved in the higher level controller where the interconnection of all the rooms and the walls are taken into consideration. The LQR controller minimizes a quadratic cost function which has two quadratic terms. One takes into account the comfort level and the other represents the control effort, i.e. the energy consumed to operate the HVAC system. There are two tuning parameters as the weight matrices for each of these two terms by which the performance of the controller can be tuned in different operating conditions. The simulation results were brought to show how much energy we could save by implementing this algorithm. It was shown that the amount of energy which can be saved depends on the level of performance that the users request from the HVAC system by assigning Q and R matrices

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