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Full Length Research Paper

Heuristic approach experience in solving mathematical problems

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Abstract

Application of procedural knowledge in solving mathematical problems requires a deep understanding of the problem posed. Heuristic approach is introduced as a tool to develop students' mathematical thinking skills. Students who have strong belief of applying heuristics approach show better experience in identifying a mathematical problem. They also show inclination in the progress of mathematical understanding which is developed through multiple strategies employed in solving mathematical problems. Thus, their effort of employing heuristics fosters strong belief on their ability in solving mathematical problems. Nevertheless, how students apply heuristic approaches successfully in solving mathematical problem is rarely focused in the teaching and learning of mathematics. This paper reports a study that investigated students' ability to develop understanding of a given mathematical problems. Their ability was observed from their work in solving the problem. Also, the participants were interviewed to examine their reflections of employing heuristics approach in the process of problem solving. Their reflections were then be related to the approaches utilized in the problem solving process. The investigation provides an insight on the mechanism of the application of heuristic approach.

Keywords: Heuristic approaches, mathematical problem, ability, belief.

INTRODUCTION

Background of the Study

Inculcating students' practice of meaningful learning is usually aimed by teachers. In conducting meaningful learning of mathematics, teachers' effort of systematic planning for creating meaningful task is required to engage the students (Coltman, Petyaeva and Anghileri, 2002). The students' practical of knowledge is always emphasized in mathematics classroom so that the values of learning mathematics are appreciated (Pathania, 2011). The practical of knowledge involving students thinking skills occurs when they are actively engaging in learning or involve in high order thinking skills. The development of thinking skills is always the focus in mathematical problem solving. Based on Polya's (1945) classical work on problem solving, he details four important steps in the thinking process in solving a problem, namely understanding the problem, devising a plan for the solution, implementing the plan and look back the solutions. There is no structured process or steps in applying the Polya's steps of solving problems since a mathematical question which exists as a problem to a student may be looked differently by another student as an easy question. Students who acquire the related mathematical knowledge may have executive control mechanisms and able to show automation of appropriate skills in solving the mathematical problems. Their ability to solve mathematical problems is built from their cognitive development (mental and conceptual development) in solving complex task with no readily accessible algorithm (Lester, 1980). Thus, in the process of solving mathematical problems, various cognitive processes are used and rationalized in reaching an acceptable solution through multiple attempts of their solutions which initiated from their initial attempt. They are believed of being able to utilize heuristics and creativity with mathematical maturity for solving either general or specific task of mathematical problems.

Students figure out and organize the steps of solutions differently. For example:

> John is visiting his father's farm. There are goats and chickens in the barnyard. He counts 20 heads, and 32 legs. How many chickens and goats are there?

If the students have never seen this kind of question before, they will have to do some reasoning about how the information given can be used to find the answers. The reasoning requires the students to develop a deep understanding by implementing the plan and looking back for examining the solution (Polya, 1962). In this example, a student may define that the total number of chicken and goat is 20 because the total number of head is 20, then assume that the number of goat is represented as x and number of chicken is represented as y, then $x+y = 20$; the number of goat is the multiple of 4 because each goat has 4 legs but for chicken is only 2, hence the relationship is $4x + 2y = 32$. On the other hand, students who have strong belief of applying heuristics approach show well experience in their discovery of mathematical problem. They also show inclination in the progress of mathematical understanding which is developed through multiple strategies employed in solving mathematical problems. Thus, their effort of employing heuristics fosters strong belief on their ability in solving mathematical problems. On the other hand, heuristic approaches are included easily through teachers' systematic planning in high engagement environment. Heuristic approaches are involved in problem solving or some other environment which needs high order thinking. The success training on the students' thinking skills intrinsically in solving mathematical problems can be evaluated from students' performance in mathematical problem solving test (Kantowski, 1977; Lukas, 1974). Nevertheless, how students can successfully apply heuristic approaches in solving mathematical problem solving is an issue. Feedback to the use of the approach can be used as a guideline for incorporating the heuristics approach in the teaching and learning process. In other words, how do the students make progress on unfamiliar or non-standard problems in focused in this study.

Hence, this paper aims to investigate students' ability to develop an understanding of the mathematical problems in the processes of solving mathematical problems. Their ability was observed from their work of solution. It also investigated how heuristic approaches can be included in teaching and learning process utilizing problem solving approach. Students' reflections of employing heuristics approach in the process of problem solving were also examined. Their reflections were related to the approaches they utilized in the solving problems. The investigation of this study provides an insight on the mechanism of the heuristic approach application.

Thus, the research questions are:

1. What are the steps of heuristic approaches utilized by students who work in groups collaboratively for solving mathematical problems?
2. What are students' beliefs of employing heuristic approaches in problem solving?

METHODOLOGY

Teaching students to use heuristics is a difficult task. In this study, the investigation for the success of employing heuristics approach was based on Polya's four steps, namely understand the problem, devise a plan, carry out the plan and look back. A group of 21 students involved in the study. The students were introduced heuristics approach in a mathematical problem solving class. During the practice of heuristics approach, the students worked in groups (G) with three members in a group. The instructor played a major role in guiding the students by posting inquiries based on the problems that the students were working on. The students were observed during their discussion in solving the problem. Heuristic approaches were highlighted in the observation. The students' solutions and their procedure of the solutions were analysed in order to match to Polya's steps of problem solving based on the heuristic approach. After the activities, the students are interviewed. They reflected on the activities in an interview. Their reflections were structured and categorized, and hence the effect of heuristic approaches in the learning was examined. The observation and interview were conducted for the following task:

Instrument

Students were assigned to solve the following problem (Figure 1). The task is a geometric question. The appropriate skills used are related to the use of coordinate geometry to represent the geometric concepts. Figure 1 shows the question for the task.

Given the square ABCD where P, Q and M are the mid points of AB, AD and CD. If AB=1m, find the area of triangle PQS.

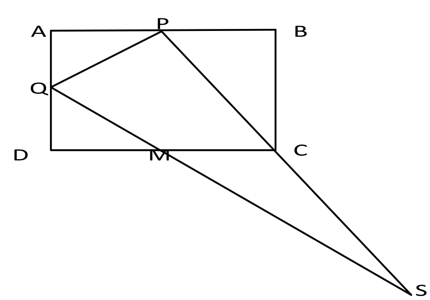

Figure 1. A mathematical problem

Data Collection

There were different topics of discussion in the class. These include calculus, algebra, statistics and geometry. This study involved the results of conducting the lesson with heuristic approach for the discussion of questions in Geometry. The activities in the classroom involved teacher's guidance and students' discussion in groups. The students' engagement was observed. The teachers' guidance aimed to engage the students' in their discussion. Teacher showed the diagram (Figure 2) to all the students as a guide to relate their previous knowledge in the discussion of the question in Figure 1.

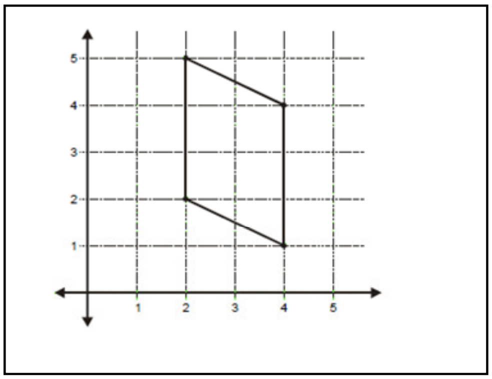

Figure 2. A geometrical shape

Inquiries were emphasised at the beginning of the use of heuristic approach. The following inquiries were involved in the discussion.

Teacher: "How can we find the area of the polygon (Figure 2) if the lengths are not given?"
One of the students (Student A) showed Figure 3 with the following feedback.
Student A: "We can count the length of sides by counting how many units have been located for the diagram."

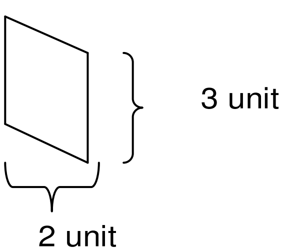

Figure 3. Diagram from student A

Student A: "Then, we find the area from $\frac{1}{2}(height)(width)$. It is $\frac{1}{2}(3\,unit)(2\,unit)$."

Student B: " The coordinates give us information of lengths, such as the base of the trapezium is measured from (4, 1) to (4,4) and the height is found from (2, 1) to (4, 1)."

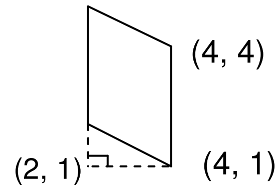

Figure 4. Diagram from student B

Student A and student B have different interpretation. Even though they presented the same answer of the area, but they indicated the height in different directions. The discussion directed more interesting conversation, such as height and width. The continuous discussion led students to imagine the diagram in Figure 1 which was located in a coordinate plane. Thus, there were solutions which indicating the I meter as 1 unit with the related coordinates as shown below (Figure 5).

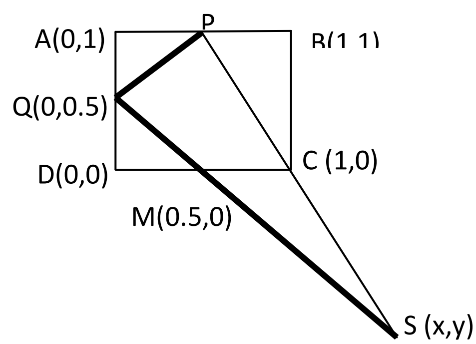

Figure 5. Coordinates for the diagram in Figure 1

RESULTS AND DISCUSSION

Following the discussion, the students found coordinate S (as shown in Figure 5). Then they found the length of the base and the height of triangle. The following solution (Figure 6) shows the solutions of finding the intersection of straight line PCS and QMS.
Then, the students easily found the length of the base from coordinates Q(0, 0.5) and S(1.5, -1), that is QS = $\sqrt{(0-1.5)^2+(0.5-(-1))^2} = \sqrt{1.5^2+1.5^2} = 2.1213$ unit.

P(0.5,1)

- Gradient for line PCS, $= \frac{1-0}{0.5-1} = \frac{1}{-0.5} = -2$

  Gradient for line QMS, $= \frac{0.5-0}{0-0.5} = \frac{0.5}{-0.5} = -1$

- Equation for line PCS,
  $y - y_1 = m_1 (x - x_1)$
  $y - 0 = -2 (x-1)$
  $y = -2x + 2$ -----(1)

  Equation for line QMS,
  $y - y_1 = m_2 (x - x_1)$
  $y - 0 = -1 (x - 0.5)$
  $y = -x + 0.5$ -----(2)

- substitute equation (1) into equation (2)
  $-2x + 2 = -x + 0.5$
  $-2x + x = -2 + 0.5$
  $x = 1.5$ -----(3)

- substitute (3) into equation (1)
  $y = -2(1.5) + 2$
  $= -1$

Therefore the **point of intersection** between both line is S(1.5, -1)

Figure 6. Finding the point of intersection

Besides that, the students easily found the length of the base from coordinates P(0.5, 1) and Q(0, 0.5), that is PQ $= \sqrt{(0.5-0)^2 + (1-0.5))^2} = \sqrt{0.5^2 + 0.5^2} = 0.7071$ unit.

Finally, they found the area of Triangle PQS as $\frac{1}{2} \times QS \times PQ = \frac{1}{2} \times 2.1213 \times 0.707 = 0.75\ unit^2$

The solutions were well presented, but the unit of the actual measurement is in meter. Thus, students' inner part of mathematical communication plays important role to enable the completion of solutions is checked throughout the process. In this case, the method of solutions from coordinates is a simulated solution from the knowledge of coordinates. The heuristics coordination of thinking is required in the reflective way of thinking.
Alternatively, students have ideas on finding the area directly from the application of finding inconsistent polygon form the following operation:

$$\frac{1}{2}\begin{vmatrix} 0 & 1.5 & 0.5 & 0 \\ 0.5 & -1 & 1 & 0.5 \end{vmatrix}$$

When they reflected to their work, they described that it was reasonable to use coordinate geometry to solve the problem by illustrating the points. The following is a sample of the reflection:

"At first we have no idea even though the question looks easy. We found difficult to get the length measurements. After the geometrical shape was introduced, we were trying to allocate the coordinates. We found that the teachers introduced us constructing a shape from the coordinates, but we did the other way round, we found the coordinates from the constructed shapes, and we should decide the first coordinate. Say that all point in the coordinate Cartesian. D is the origin (0,0) and we knew that the angle of PQS is 90°. First we found the point of intersection, S, between straight line PCS and straight line QMS. By using the Pythagoras Theorem to find the length of PQ and QMS, we should get the area of triangle PQS…." (G1).

When the students were asked to reflect about their group work in using the heuristics approach, they revealed that

(1) They need more activities and more practice. They expressed that:

"Practice is important. We always ask ourselves about the requirement of the questions through diagram, tables and etc. Solving mathematical problem is not difficult. We believed that more exploration and understand heuristics approach will save us." (G1)

"We always cannot answer problem solving questions. We cannot see the hidden points. After we gone through more questions with practices, We are more confident. We always shorten the question. We find that simplify the question is important." (G2)

(2) Going through the first step and second step in the Polya's processes of solving problems are the most difficult steps. They uncovered that:

> " Problem solving is difficult. But, it is important because it involves mathematical thinking. The first step is the most important step in answering. Heuristic is communication of mathematical problem. But, we believe that their steps on define a problem and then devise a plan took a lot of time." (G3)

CONCLUSION

The given activity had enhanced students' knowledge on some other topics, such as the topic of geometry. Even though students experienced some difficulties in applying heuristics approach, they found that frequent practice of the approach will make solving different problems easy. The students were able to solve a given problem which was non-routine using heuristic approach. The engagement of group activity had also improved their confidence in the process of solving the problem. Their active role of working on the problems in order to find the solutions is a good sign of exploring mathematical problems (Fuson et al., 1997). The active engagement has changed students' beliefs about mathematics they perceived earlier that the problems can be solved using a formula. It was found in the heuristic application students put more emphasis understanding the problems (as indicated in the first and second step of Polya's problem solving process). Such finding corresponds to studies by a number of researchers (e.g., Carpenter et al., 1998; Cobb et al., 1991; Verschaffel and De Corte, 1997) who advocate that it is possible to change students' beliefs about mathematics through the process of solving problem. In addition, the study construes that teacher's guidance created a chance for students to explore the solutions with their own strategies.

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