

Full Length Research Paper

Getting fresh water from a still lagoon and with no moving parts

¹Ronald L. Huston and ²John M. Dobbs

¹Department of Mechanical Engineering, University of Cincinnati. P.O. Box 210072, Cincinnati, OH 45221-0072

²Ryalside Institute, 12 Cressy Street, Beverly, MA 01915

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In this paper we establish the feasibility of extracting clean water from a lake or a lagoon and then depositing it on adjacent arid land. The entire process is empowered using only the daytime/nighttime temperature variation. To obtain this water transformation and irrigation we use a large greenhouse type structure covering both a portion of the water and the adjacent shore land. The solar energy of the sun shining through the greenhouse heats the water producing a high temperature air/water-vapor mixture. Low evening temperature then produces a heavy dew upon the land. The paper provides the underlying thermodynamic analysis for the process, together with a brief review of the essential concepts including: mass; weight; pressure; partial pressure; volume; specific volume; temperature; molecular hypotheses; ideal gases; mixtures; saturation; evaporation; relative humidity; specific humidity; and dew point. We conclude with a series of illustrative computations validating the process. The results show that water condensate increases linearly with the greenhouse volume and with the humidity, but exponentially with the greenhouse temperature.

Keywords: Condensation, distillation, humidity, water resources.

INTRODUCTION

Our intent in this paper is to establish the feasibility and practicality of obtaining fresh water from humid greenhouse air. A "greenhouse" simply refers to an enclosure with a translucent surface cover transmitting sunlight energy to the interior. We envision a large enclosure partially covering a natural body of calm water, such as a lake or a lagoon, in a relatively stable, but warm arid climate.

During the day, the sunlight shining through the greenhouse heats and evaporates the water producing a humid air environment. Then during the cooler evening and nighttime the water vapor condenses. If a portion of the greenhouse covers the shore, the condensate will produce dew on the land. That is, the greenhouse will in effect collect water vapor from the lagoon in the day and deposit a portion of it as dew on the dry land during the night.

The principal issue is: How much water can reasonably be obtained on a daily basis? In the following paragraphs we answer this question, and in the process, we develop a procedure for calculating the amount of water produced for any given environmental condition.

We envision using the analysis to design, develop, and fabricate a greenhouse sufficiently large to irrigate arid soil without the use of energy sources nor driving devices, aside from the sun. The objective is to produce fresh water in an arid climate using only daytime solar heating and evening cooling.

The content of what follows is divided into five parts with the first of these providing a thermodynamic review, establishing a basis for the subsequent analyses. In this review, we briefly summarize the concepts, conventions, and notations of and for mass, weight, pressure, volume, and temperature. We consider the physics of ideal gasses, water vapor, phase changes, the gas constants, and the Avogadro hypothesis. We conclude the review with a short description of partial pressures, Dalton's model, humidity, and the dew point phenomenon.

*Corresponding Author E-mail: ron.huston@uc.edu;
Tel: 1(513) 556-6133, fax: (513) 556-3390

The thermodynamic review is intended to be simply a quick reference for readers evaluating the water production concepts.

The second and third parts illustrate the quantification of water production, providing a description of the way various parameters affect the production.

The fourth part establishes the feasibility of irrigation of arid lands, and the final part is a discussion with concluding remarks.

METHODS

Water Heating and Cooling: Phase Changing

Consider the familiar process of boiling water: Specifically, consider a container of water at room temperature and at standard atmospheric pressure. By heating the water and raising its temperature to the boiling point and beyond, the water is vaporized, becoming steam, or "superheated vapor". During boiling the water undergoes a liquid-to-gas phase change.

The process is readily reversed by cooling the water vapor, causing it to condense, undergoing a gas-to-liquid phase change.

While boiling and condensation is a commonly known and understood process, what is less well known is that water can also be vaporized without heating -- but instead by simply lowering the pressure. That is, we can obtain a liquid-to-gas phase change at room temperature by sufficiently lowering the pressure. For example, at 70°F (21.1°C) water will boil if the pressure is reduced to 0.3631 psi (2.503 kPa).

The evaporation process may be visualized microscopically as energetic water molecules at the liquid surface escaping the pull of the liquid and moving as free molecules into the air.

Alternatively, as the temperature is lowered in an air-water vapor mixture, and as the mixture is cooled by heat energy leaving the mixture, the kinetic energies of the freely moving water vapor molecules are correspondingly reduced. The slower moving water molecules then coalesce and form condensate droplets.

Suppose now that a lagoon water surface under a warming greenhouse cover evaporates and produces a humid, nearly saturated air-water vapor mixture in the greenhouse. Suppose further that a portion of the greenhouse extends over adjacent arid shore land. Then if the lagoon end of the greenhouse is enclosed and the land end has an opening, the expanding humid lagoon air will be directed by the greenhouse to float over the shore land, and beyond. Evening cooling will then deposit condensed dew onto the land.

In the following sections we provide a brief thermodyna-

mic review enabling us to quantify the amount of water deposited on the land.

Thermodynamic Considerations

Notation

atm	atmosphere
ft	foot, feet
g	gram, gravity
Hg	mercury
J	Joule (Newton-meter)
K	Kelvin (degrees)
kg	kilogram
kPa	kilo Pascal
lb	pound
lb _f	pound force
lb _m	pound mass
m	meter, mass
M	molar mass
M _{air}	approximate equivalent molar mass of air
m _{air}	mass of air in an air/water vapor mixture
m _g	mass of water vapor in saturated vapor
M _{H₂O}	molar mass of water vapor
m _{H₂O}	mass of water vapor in an air/water vapor mixture
mm	millimeters
N	Newton
p	pressure (absolute)
Pa	Pascal (Newtons per square meter)
p _{air}	partial pressure of air in an air/water vapor mixture
p _{H₂O}	partial pressure of water vapor in an air/water vapor mixture
psi	pounds per square inch
R	ideal gas constant
R _{air}	gas constant for air
R _{H₂O}	gas constant for water vapor
s	second, specific entropy
sec	second
s _f	specific entropy of saturated liquid for water vapor
s _g	specific entropy of saturated vapor for water vapor
SI	International System of Units
T	temperature (absolute)
T _{dew}	dew point temperature
v	specific volume
V	volume
v _f	saturated liquid specific volume for water vapor
v _g	saturated vapor specific volume for water vapor
v _{H₂O}	specific volume of water vapor in an air/water vapor mixture
W	weight
x	quality
°C	degrees Celsius (Centigrade)

$^{\circ}\text{F}$	degrees Fahrenheit
$^{\circ}\text{R}$	degrees Rankine
ϕ	relative humidity
S	mass density
g	specific gravity
α	humidity ratio (specific humidity)

Thermodynamics is primarily an analysis of the behavior of gasses at varying temperatures and pressures. To a lesser extent thermodynamics is a corresponding analysis of the behavior of liquids and solids and the transitions between solid, liquid, and gas (vapor) phases. In thermodynamic analyses the analysts typically employ a series of scalar variables to quantify the behavior of a given system of gasses, liquids or solids. The most common of these variables are: temperature, pressure, volume, heat, work, energy, and entropy. Thermodynamic analyses then lead to quantitative relations between the variables and limitations on processes. Fortunately for our purposes we need only consider a few of these variables and only to a limited extent.

In the following paragraphs we provide a brief review of elementary thermodynamic concepts useful in our envisioned water recovery process.

Mass and weight

The mass of a substance or an object is a measure of the amount of material making up the substance or object. The mass is also a measure of the resistance of a substance or an object to being accelerated by a force, where intuitively, a force is simply a "push" or a "pull".

The weight of a substance or an object is a measure of the gravitational attraction toward the center of the earth when the substance or object is near the surface of the earth. Unfortunately in the English (American) system of units weight is generally used as a measure of the amount of matter making up a substance or an object. There is thus an occasional confusion between mass and weight -- particularly in thermodynamic analyses. But the relationship is simple: If an object has a mass m and a weight W then

$$W = mg \quad (1)$$

where g is the acceleration of a falling body due to gravity. In the English system g is approximately 32.2 feet per second squared (ft/sec^2), and in the International system (SI) g is approximately 9.8 meters per second squared (m/s^2).

In the English system, in kinematic/dynamic analyses, the fundamental unit of force is the "pound", written as: lb , and the fundamental unit of mass is the slug. In free space a force of one pound will accelerate a mass of one slug, one foot per second squared. That is

$$1 \text{ lb} = 1 \text{ slug ft/sec}^2 \quad (2)$$

Alternatively, from Eq. (1) an object with a mass of one slug weighs 32.2 lb.

In thermodynamic analyses, however, accelerations are seldom of interest. (Hence, the occasional name: "thermostatics".) Traditionally, in the English system, thermodynamicists have used the fundamental unit of mass as the "pound mass" which is simply the mass of a substance, or an object, having a weight of one pound. Then to distinguish between "pound force" and "pound mass" the symbols lbf and lb_m are used respectively. Thus,

$$1 \text{ slug} = 32.2 \text{ lb}_m \quad (3)$$

In the currently widely used international system (SI), confusion is avoided by using different and distinct units for mass and weight: The fundamental mass unit in the SI system is the kilogram (kg) and the fundamental unit of force is the Newton (N). In free space one Newton will accelerate one kilogram one meter per second squared. That is,

$$1 \text{ N} = 1 \text{ kg m/s}^2 \quad (4)$$

From Eq. (1) a mass of 1 kg has a weight of 9.8 N.

We can readily develop relatively accurate conversion factors between the two unit systems by using Eq. (1) and the exact relation that one inch ($1/12 \text{ ft}$) is 2.54 centimeters (0.01 m). With its various tables and listings, provides a summary of useful conversions.

Briefly, we have the following relations:

$$\begin{aligned} 1 \text{ kg} &= 2.225 \text{ lb}_m, \\ 1 \text{ lb}_m &= 0.445 \text{ kg}, \\ 1 \text{ slug} &= 32.17 \text{ lb}_m \\ 1 \text{ N} &= 0.225 \text{ lb}_f, \\ 1 \text{ lb}_f &= 4.448 \text{ N} \end{aligned} \quad (5)$$

Pressure

Pressure is simply a measure of force per unit area -- or "normalized force". For our purposes, pressure may be viewed as the magnitude of a uniform force system applied perpendicular (or normal) to a surface divided by the area of the surface. Pressure is thus a compressive (or "pushing") force and regarded as positive in thermodynamics (absolute pressure).

The English units for pressure are pounds per square inch (lb/in^2 or psi). The SI units are Newtons per square meter [N/m^2 or Pascals (Pa)]. On occasion, if pressure is measured by a manometer the units may be either inches or millimeters of mercury. The conversions are:

$$\begin{aligned} 1 \text{ psi} &= 6.895 \text{ kPa} = 2.036 \text{ in Hg} = 51.714 \text{ mm Hg} \\ \text{and} \quad 1 \text{ Pa} &= 1.450 \times 10^{-4} \text{ psi} = 2.953 \times 10^{-4} \text{ in Hg} = 7.501 \times 10^{-3} \text{ mm Hg} \end{aligned} \quad (6)$$

The "standard atmospheric pressure" (a traditionally useful concept in thermodynamic analyses) is assigned the values: 14.6959 psi, or 101.325 kPa (Van Wylen and Sonntag, 1965, 1978), or 1 atmosphere (1 atm or 1 bar). For our purposes we can use the approximate values:

$$1 \text{ atm} = 14.7 \text{ psi} = 101 \text{ kPa} = 1 \text{ bar} \quad (7)$$

Values of pressure above (or below) atmospheric pressure are called "gage" pressures. Values of pressure, including

the atmospheric pressure -- that is, with respect to zero pressure (a complete vacuum) -- are called "absolute" pressures. Thermodynamic analysts usually employ absolute pressure.

Finally, in mixtures of gasses the pressure may be viewed as the sum of component pressures (or "partial pressures") of the individual gasses. We discuss partial pressure in a later section.

Volume, Specific Volume

Volume is a measure of the space occupied by a substance or a gas. In the English system the typical units are cubic feet (ft^3). The SI units are cubic meters (m^3).

"Specific volume" is the volume per unit mass, or normalized volume. In the English system the typical units are cubic feet per pound mass (ft^3/lb_m). The SI units are cubic meters per kilogram (m^3/kg).

Specific volume is simply the reciprocal of mass density.

The conversion units are:

$$\begin{aligned} 1 \text{ ft}^3 &= 0.0283 \text{ m}^3 & 1 \text{ m}^3 &= 35.31 \text{ ft}^3 \\ 1 \text{ ft}^3/\text{lb}_m &= 0.0624 \text{ m}^3/\text{kg} & 1 \text{ m}^3/\text{kg} &= 16.02 \text{ ft}^3/\text{lb}_m \end{aligned} \quad (8)$$

Temperature

Temperature is a measure of molecular kinetic energy in a substance or gas with higher temperature values with higher molecular energy values. There are both absolute and relative temperature scales, in both the English and SI systems. With the absolute scales a temperature of zero refers to zero molecular energy. The relative temperature scales have values related to the phase changes (freezing and boiling) of water.

Both the English and SI systems employ the temperature unit of degree, but the SI degrees are larger, being 9/5 the size of the English degrees. In the English system the absolute temperature is measured using the Rankine scale with the degrees represented as: $^{\circ}\text{R}$. The relative English temperature scale is the Fahrenheit scale with the degrees represented as: $^{\circ}\text{F}$. The absolute temperature 459.67°R , corresponds to 0°F . Boiling occurs at 212°F , and freezing at 32°F .

In the SI system the absolute temperature is measured using the Kelvin scale with the degrees represented simply as: K. The relative SI temperature scale is the Celsius scale (formerly called Centigrade scale) with the degrees represented as: $^{\circ}\text{C}$. The absolute temperature: 273.15K corresponds to 0°C , at water freezing. Boiling occurs at 100°C . The conversions are (Potter and Somerton, 1993):

$$\begin{aligned} (^{\circ}\text{F}) &= (9/5)(^{\circ}\text{C}) + 32 & ^{\circ}\text{C} &= (5/9)(^{\circ}\text{F}) - 32 \\ (^{\circ}\text{R}) &= (^{\circ}\text{F}) + 459.67 & (\text{K}) &= (^{\circ}\text{C}) + 273.15 \end{aligned} \quad (9)$$

Moles, Molar Mass, and Avogadro Hypothesis

A "mole" is a quantity, an amount, or a number of atoms or molecules of a substance whose mass, measured in grams is numerically equal to the atomic mass value of the substance. For example, a carbon atom has an atomic mass of 12.01 and a mole of carbon atoms has a mass of 12.01 grams (12.01g). Similarly, oxygen has an atomic mass of 16 and a molecular (O_2) mass of 32. Then a mole of oxygen gas has a mass of 32g.

The number of entities in a mole is: 6.02×10^{23} (Avogadro's number) (Silberberg, 2007). A mole is thus a numerical quantity, such as a "dozen", but sufficiently large that it can model atomic entities in a macroscopic way. It is analogous to the use of a "light year" to describe astronomical distances. (Interestingly 10 moles of kilograms is approximately equal to the mass of the earth (Holbrow, Lloyd, and Amato, 1999).)

The "molar mass" M , of a substance is the mass in grams of a mole of the substance (Silberberg, 2007). For example, the molar mass of a water molecule $\text{M}_{\text{H}_2\text{O}}$ is 18.08 g/mole. The molar mass of air (a mixture of gasses) M_{air} , is approximately 28.95 g/mole.

"Avogadro's hypothesis" is that equal volumes of any gas, at the same pressure and temperature, contain identical numbers of molecules. That is, the number of neutrons, protons, and electrons of an atom of a gas molecule does not affect the volume of a mole of a gas. This implies that the "size" of a molecule does not affect the molar volume. Alternatively, in a gas, there is a relatively large separation between individual molecules of the gas.

At a temperature of 0°C (273.15 K) and at a pressure of 1 atm, the volume of an ideal gas ("standard molecular volume") is 22.4 liters or 10^{-3} m^3 (Silberberg, 2007).

Ideal Gas Law, Ideal Gas Constant

An "ideal" gas satisfies the "ideal gas law" which may be expressed in a variety of forms such as (Holbrow, Lloyd, and Amato, 1999; Potter and Somerton, 1993; Silberberg, 2007; Taylor, Zafiratos, and Dubson, 2004; Van Wylen and Sonntag, 1965, 1978) :

$$pv = RT, \quad pV = mRT, \quad p = \rho RT, \quad pV = n\bar{R}T \quad (10)$$

where p is the pressure (absolute). T is the absolute temperature, V is the volume, v is the specific volume, m is the mass, ρ is the mass density, R is the universal gas constant, \bar{R} is the gas constant for the particular gas being considered, and n is the number of moles of the gas. In the English and SI systems the universal gas constant has the values:

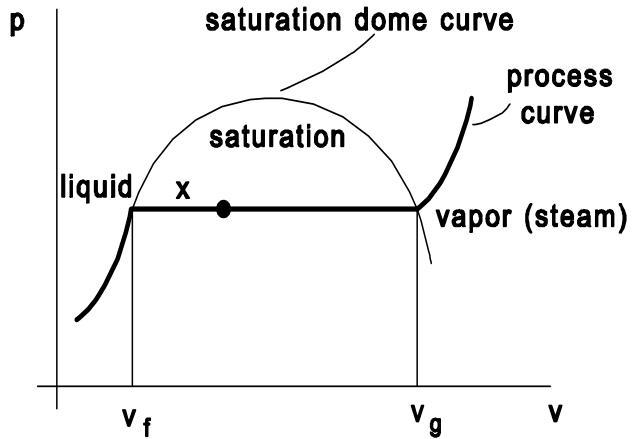


Figure 1. Pressure-Specific Volume (p-v) Diagram.

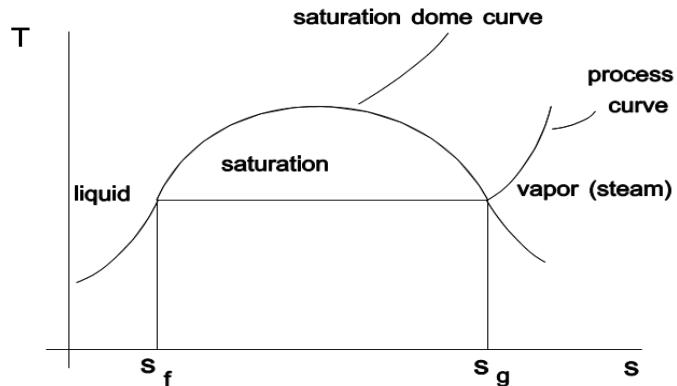


Figure 2. Temperature-Specific Entropy (T-s) Diagram.

$$\bar{R} = 1545 \text{ ft lb}_f / \text{lb}_m \text{ mole } ^\circ\text{R} \quad (11)$$

and

$$R = 8.314 \text{ Nm/g mole K} \quad (12)$$

The gas constant R for any given ideal gas may be obtained from the universal constant by the simple expression:

$$R = \bar{R}/M \quad (13)$$

where, as before, M is the molar mass.

For example, for oxygen, O₂, (a near ideal gas) the molar mass is: 32. Then the oxygen gas constant: is:

$$R_{O_2} = 1545 / 32 \text{ ft lb}_f / \text{lb}_m \text{ mole } ^\circ\text{R} = 48.28 \text{ ft lb}_f / \text{lb}_m \text{ mole } ^\circ\text{R} \quad (14)$$

and

$$R_{O_2} = 8.314 / 32 \text{ Nm/g K} = 0.260 \text{ kJ/kg K} \quad (15)$$

where the unit J (called a "Joule") is a Newton-meter (Nm).

In our analysis we assume that the behaviors of air and water vapor, at atmospheric conditions, may be reasonably approximated by the ideal gas law. In this regard the gas constants for air (R_{air}) and water vapor are

found to be:

$$R_{air} = 53.35 \text{ ft lb}_f / \text{lb}_m \text{ mole } ^\circ\text{R} = 0.287 \text{ kJ/kg K} \quad (16)$$

and

$$R_{H_2O} = 85.76 \text{ ft lb}_f / \text{lb}_m \text{ mole } ^\circ\text{R} = 0.462 \text{ kJ/kg K} \quad (17)$$

Liquid/Vapor Phases Changes for Water

A pan of boiling water is the familiar illustration of a substance (the water) undergoing a phase change. During this phase change the liquid water and the water vapor coexist at a fixed pressure and temperature. At standard atmospheric pressure: 14.7 psi or 100 kPa, the boiling temperature is: 212°F or 100°C. During the phase change the water vapor is said to be "saturated". As the pressure is lowered the boiling temperature is also lowered.

The behavior of water at pressures and temperatures above and below saturation, and at saturation itself, may be visualized by pressure-volume (p-V) and temperature-entropy (T-s) graph diagrams as in Figures 1 and 2.

In the graph of Figure 1 the abscissa is the specific volume v , and the bold curve in the figure describes the relationship between the pressure and the specific volume during a typical phase change process of the water substance. The horizontal line portion of the bold curve represents the saturation state where liquid and vapor coexist. At the left end of this line the substance is all liquid, and known as "saturated liquid," with specific volume v_f . At the right end the substance is all vapor, and known as "saturated vapor" with specific volume v_g . At an intermediate point, the substance is a mixture of liquid and vapor. The fractional distance x along the line is known as the "quality" of the mixture.

Finally, in Figure 1. the region beneath the "dome curve" is the saturated (mixed liquid/vapor) region. The regions to the left and right of the dome curve are called "compressed liquid" and "saturated vapor" (steam) regions respectively.

In the graph of Figure 2 the ordinate is the absolute temperature T and the abscissa is the specific entropy (entropy per unit mass) s . "Entropy", as with "energy", is simply a convenient thermodynamic variable used to describe the state of a substance and the behavior of a process. Entropy is a measure of the complexity, chaos, and disorder of the movements of the water molecules. "Complexity" is a description of the compressed liquid up to the point of boiling (saturation) where "chaos" occurs. Beyond chaos there is increasing "disorder".

The curves and regions in the T-s diagram Figure 2 are similar and analogous to those of the p-v diagram of Figure 1. Observe that, as with the pressure, the temperature is constant in the saturation (phase-changing) region.

In the thermodynamic analysis of a process between two states (say 1 and 2) the area under the process curve in a p-v diagram represents the work done, W_2 by the water during the process of changing from state 1 to state 2. Similarly, the area under the curve in a T-s diagram represents the heat absorbed by the water during the process from state 1 to state 2, Q_2 .

(Specific entropy has the units of energy per unit mass-degree.)

Finally, we see from the ideal gas law [Equations (10)], and from empirical data on the saturation states of liquid/vapor water (Keenan et al., 1969a, b); that the saturation pressures and temperatures are directly related. This means that if the saturation pressure is sufficiently low, the saturation temperature can occur at sea level atmospheric temperatures.

The Concept of Partial Pressures in a Gas Mixture -- Dalton's Model

Imagine a rigid container filled with a gas. According to the kinetic theory of gasses (Taylor, Zafiratos & Dubson 2004; Silberberg 2007), the "pressure" of the gas is the same as the pressure (force per unit area) on the interior walls of the

container. The interior wall pressure in turn is due to gas molecules colliding with and then rebounding from the walls. The greater the amount of gas molecules in the container, the greater the number of collision/rebounds with the interior walls, and thus the greater the pressure.

Suppose now that the gas in the container is a mixture of gasses -- such as air and water vapor. Suppose further that the mixture is uniform so that with say an air/water vapor mixture the water molecules are uniformly dispersed within the various of the air gasses molecules. The kinetic theory assumes that even though the gasses are under pressure there are still great distances (from an atomic perspective) between individual molecules. This in turn means that with each gas being uniformly dispersed, each gas occupies the entire volume of the container. Moreover, with the uniform dispersal of mixed gasses, the molecule wall collision and rebounding occurs with each of the constituent gasses.

Dalton's law (Van Wylen & Sonntag 1978; Potter and Somerton 1993; Cengell & Boles 1994; Taylor, Zafiratos, and Dubson 2004; Silberberg 2007); states that the forces occurring due to the molecular wall collisions and rebounds are additive and thus create "partial pressures" whose sum is the overall pressure within the container. Specifically, if the container has a mixture of air and water vapor the pressure p in the container may be expressed as:

$$p = p_{\text{air}} + p_{\text{H}_2\text{O}} \quad (18)$$

where P_{air} and $P_{\text{H}_2\text{O}}$ are the partial pressures of the air and water vapor.

Observe from Equation (18) that if there are considerably more air molecules than water vapor molecules, the partial pressure of the air P_{air} will considerably exceed the partial pressure of the water vapor. The relatively low water vapor pressure will then enable the occurrence of the existence of low saturation temperatures.

Dew Point and Dew Point Temperature

If an air/water vapor mixture is cooled at a constant pressure (as in the evening and into the nighttime), the water vapor will begin to condense, changing phase to liquid. The state where condensation begins is called the "dew point" and the temperature where it occurs is called the "dew point temperature".

Figures 3 and 4 illustrate the concepts: Imagine the water vapor of an air/water-vapor mixture to be in the superheated state at point A. Let the mixture then be cooled at constant pressure to point B which is at the intersection of the constant pressure curve (an "isobar") and the saturation dome curve. Point B is the dew point and the saturation temperature at the dew point is called "the dew point temperature".

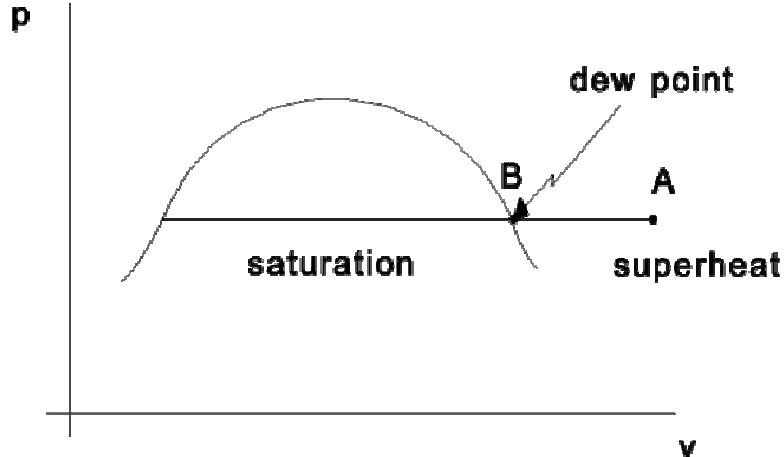


Figure 3. Dew Point on a p-v Diagram for Cooling at Constant Pressure

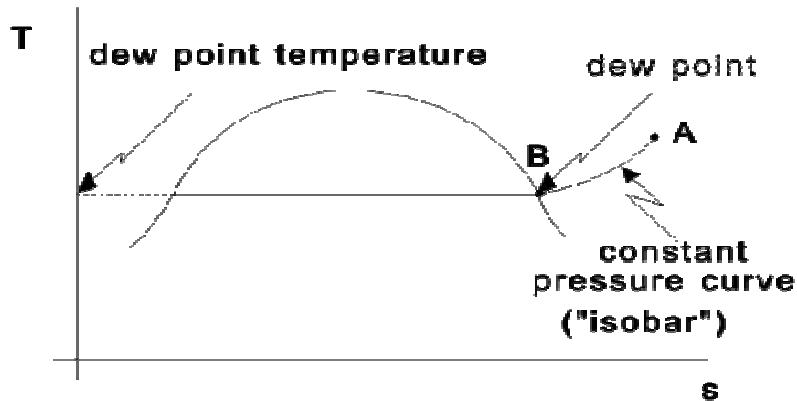


Figure 4. Dew Point and Dew Point Temperature on a T-s Diagram for Cooling at Constant Pressure

Humidity

"Humidity" is a measure of water content and/or water concentration in an air/water-vapor mixture. There are two humidity measures: 1) "relative humidity" and 2) "humidity ratio".

Relative humidity is the percentage of mass of water vapor in an air/water vapor mixture relative to the maximum possible mass of water vapor that the air can hold at the same pressure and temperature. That is,

$$\phi = m_{H_2O} / (m_{H_2O})_{\max} \quad (19)$$

For a given volume V of the air/water-vapor mixture the specific volume v_{H_2O} of the water vapor is simply the ratio of the water vapor mass to V . That is,

$$v_{H_2O} = V / m_{H_2O} \quad (20)$$

When the mass of water vapor present is at its maximum level (m_{H_2O})_{max}, the specific volume of the water vapor will

be at a minimum (at the dew point) and have the value: v_g (saturated water vapor). That is

$$v_g = V / (m_{H_2O})_{\max} \text{ or } (m_{H_2O})_{\max} = V / v_g = m_g \quad (21)$$

Then by substituting into Equation (19) the relative humidity is given by:

$$\phi = v_g / v_{H_2O} = m_{H_2O} / m_g \quad (22)$$

The "humidity ratio" w (aka "specific humidity") is the ratio of the mass of water vapor to the mass of air in an air/water vapor mixture. That is,

$$\omega = m_{H_2O} / m_{air} \quad (23)$$

By eliminating m_{H_2O} between Equations (22) and (23) we see that the relative humidity and the humidity ratio are related by the simple expression:

$$\omega = (m_g / m_{air}) \phi \quad (24)$$

Recall from Equations (10), the "ideal gas law", that $pV = mRT$ (25)

Table 1. Unit Abbreviations and Symbols.

Unit Name	Abbreviation/Symbol
Atmosphere	atm, bar
Degrees Celsius (Centigrade)	$^{\circ}\text{C}$
Degrees Fahrenheit	$^{\circ}\text{F}$
Degrees Rankine	$^{\circ}\text{R}$
Foot, Feet	ft
Gram	g
Inch, Inches	in
Joule	J, Nm
Kelvin (degrees Kelvin)	K
Kilo Joules	kJ
Kilo Pascal	kPa
Kilogram	kg
Mercury	Hg
Meter	m
Millimeters	mm
Newton	N
Newtons per square meter	N/m ² , Pa
Pascals	Pa
Pound force, Pound mass	lb _f , lb _m
Pounds per square inch	lb/in ² , psi
Second	sec, s
Slug	slug

Table 2. Unit Conversions.

	Given	To obtain	Multiply by
1. Energy	ft lb _f	J, Nm	1.356
	ft lb _f	kJ	1.356×10^{-3}
	J, Nm	ft lb _f	0.7376
	kJ	ft lb _f	7.376×10^{-2}
2. Force	lb _f	N	4.448
	N	lb _f	0.2248
3. Length	ft	m	0.03048
	in	mm	25.4
	m	ft	3.280
	mm	in	3.937×10^{-2}
4. Mass	lb _m	g	4.536×10^{-2}
	lb _m	kg	0.4536
	lb _m	slug	3.105×10^{-2}
	slug	lb _m	32.2
	slug	kg	14.59
	g	lb _m	2.205×10^{-3}
	kg	slug	6.852×10^{-2}
	kg	lb _m	2.205

Table 2. Continue.

5. Pressure	lb _f /in ² , psi	N/m ² , Pa	6.895 x 10 ³
	lb _f /in ² , psi	in Hg	2.036
	lb _f /in ² , psi	mm Hg	51.71
	N/m ² , Pa	lb _f /in ² , psi	1.450 x 10 ⁻⁴
	N/m ² , Pa	mm Hg	7.498 x 10 ⁻³
	N/m ² , Pa	in Hg	2.952 x 10 ⁻⁴
	atm	lb _f /in ² , psi	14.7
	atm	in Hg	29.92
	atm	N/m ² , Pa	1.013 x 10 ⁵
6. Specific Volume	ft ³ /lb _m	m ³ /kg	6.243 x 10 ⁻²
	m ³ /kg	ft ³ /lb _m	16.018
7. Temperature	°F, °R	°C, K	0.5555
	°C, K	°F, °R	1.8
8. Volume	ft ³	m ³	2.832 x 10 ⁻²
	m ³	ft ³	35.314

Table 3. Thermodynamic Constants.

Name/Symbol	Value
Avogadro number	6.02 x 10 ²³
Ideal gas constant, \bar{R}	1545 ft lb _f /lb _m mole °R = 8.314 Nm/g mole K
Gas constant for water vapor, R _{H₂O}	85.76 ft lb _f /lb _m 0R = 0.462 kJ/kg K
Gas constant for air, R _{air}	48.28 ft lb _f /lb _m 0R = 0.287 kJ/kg K
Molar mass of water vapor M _{H₂O}	18.015
Approximate equivalent molar mass of air, M _{air}	28.97
Standard atmospheric pressure, p ₀	14.7 psi = 101.3 kPa

Table 4. Temperature Values at Various Conditions and States.

Temperature scale	Absolute zero	Water freezes	Water Boils	Room Temperature
Fahrenheit, °F	-459.67	32	212	72
Celsius, °C (formerly Centigrade)	-273.15	0	100	22.2
Rankine °R	0	491.67	671	531
Kelvin	0	273.15	373.15	295.37

Then for a container or region with a fixed volume V and temperature T, we have the expressions:

$$P_{H_2O}V = m_{H_2O}R_{H_2O}T, \quad p_gV = m_gR_{H_2O}T, \quad p_{air}V = m_{air}R_{air}T \quad (26)$$

Where as before P_{H₂O}, P_g and P_{air} are the partial pressures of the water vapor, the saturated water vapor, and the air respectively. By dividing the first two expressions we

have:

$$P_{H_2O}/p_g = m_{H_2O}/m_g = \phi \quad (27)$$

where the last equality is given by Equation (22).

Next, by dividing the last two expressions of Equation (26), we have

$$p_g/p_{air} = (m_g/m_{air})(R_{H_2O}/R_{air}) \text{ or } m_g/m_{air} = (p_g/P_{air})(R_{air}/R_{H_2O}) \quad (28)$$

Finally by substituting this last result into Equation (24) we have

$$\omega = \left(R_{H_2O} / R_{air} \right) \left(p_g / p_{air} \right) \phi = 0.622 \left(p_g / p_{air} \right) \phi \quad (29)$$

where the numerical result is obtained using the values of R_{air} and R_{H_2O} from Equations (16) and (17).

Units, Thermodynamic Constants, and Conversion Factors

In this final review section we provide a series of tables useful in making computations and thermodynamic analyses.

Illustrative Analyses

In the following paragraphs we provide a number of analyses with numerical results intended to provide a guide for the geometric design of the envisioned greenhouse. We assume the structure will cover a vast supply of lagoon water in a stable climate with reasonably predictable day/night temperature variations.

The objective of the computations is to determine the amount of water condensed during nighttime cooling. The procedure is relatively simple in view of the foregoing thermodynamic review.

We assume a given volume V of the greenhouse interior, at a given temperature T and relative humidity ϕ . Then at atmospheric pressure (14.7 psi or 101.3 kPa) we compute the mass of condensed water when the temperature is lowered to a given nighttime temperature.

From the given daytime conditions we can readily compute the mass of water vapor present during the day. Then knowing the nighttime temperature we can similarly compute the mass of water vapor that can coexist with the air at the reduced temperature. The difference is the mass of water condensed.

We consider five cases to illustrate the procedure.

Case 1. Given; Greenhouse volume, $V = 1000 \text{ ft}^3$; pressure, $p = 14.7 \text{ psi}$; relative humidity, $\phi = 75\%$; temperature, $T = 80^\circ\text{F}$

Find. The water condensed when the air/water vapor mixture is cooled to 50°F

Analysis. During the day we have:

$$80^\circ\text{F} = 539.67^\circ\text{R} = T_{day} \quad (30)$$

From steam tables (Keenan et al. 1969) if the water vapor is in the saturation state, the saturation temperature tables at 80°F yield

$$p_g = 0.5073 \text{ psi} \quad (31)$$

(Observe how low p_g is.)

From Equation (27) the partial pressure of the water vapor is then

$$p_{H_2O} = \phi p_g = (0.75)(0.5073) = 0.3805 \text{ psi} \quad (32)$$

Using the ideal gas law (Equation (10) we see that the mass of the water vapor m_{H_2O} in the daytime mixture is then:

$$m_{H_2O} |_{day} = \frac{p_{H_2O} V}{R_{H_2O} T_{day}} = \frac{(0.3805)(1000)(144)}{(85.76)(539.67)} = 1.184 \text{ lb}_m \quad (33)$$

where from Equation (17) 85.76 is the gas constant for water in $\text{ft lbf/lbm}^\circ\text{R}$, and 144 is a conversion from ft^2 to m^2 .

Consider now the nighttime conditions: From Figures 3 and 4 and the steam tables we see (via interpolation) that at a pressure of 0.3805 psi the dew point temperature T_{dew} is $T_{dew} = 71.37^\circ\text{F}$ (34)

Since the dew point temperature (71.37°F) is considerably higher than the given nighttime temperature (50°F), the relative humidity of the nighttime air/water vapor mixture is 100%. Also at nighttime the temperature is

$$50^\circ\text{F} = 509.67^\circ\text{R} = T_{night} \quad (35)$$

From the saturation temperature steam tables (Keenan et al. 1969a) at 50°F , p_g is:

$$p_g = 0.17803 \text{ psi} \quad (36)$$

From Equation (27), at 100% humidity, the partial pressure of the water vapor is:

$$p_{H_2O} = \phi p_g = (1.0)(0.1780) = 0.1780 \text{ psi} \quad (37)$$

Using the ideal gas law [Equation (10)] we see that the mass of water vapor in the nighttime mixture is:

$$m_{H_2O} |_{night} = \frac{(0.1780)(1000)(144)}{(85.76)(509.67)} = 0.5865 \text{ lb}_m \quad (38)$$

Finally, by subtracting the water vapor masses between the daytime and the nighttime (Equations (33) and (38)), we obtain the mass of the condensate as:

$$\text{condensate} = 1.184 - 0.5865 = 0.5975 \text{ lb}_m \quad (39)$$

Case 2

Repeat the analysis of Case 1 with the same conditions except with a greenhouse volume: $V = 2000 \text{ ft}^3$.

Analysis

In reviewing the analysis of Case 1 we see that the volume affects the computations only in Equations (33) and (38), for

the masses of water vapor in the day and nighttime air/water-vapor mixtures. With the volume at 2000 ft³ these masses become:

$$m_{H_2O} |_{day} = \frac{(0.3805)(2000)(144)}{(85.76)(539.67)} = 2.3677 \text{ lb}_m \quad (40)$$

and

$$m_{H_2O} |_{night} = \frac{(0.1780)(2000)(144)}{(85.76)(509.67)} = 1.1728 \text{ lb}_m \quad (41)$$

By subtracting the nighttime mass from the daytime mass we see that the condensate is:

$$\text{condensate: } 2.3677 - 1.1728 = 1.1949 \text{ lb}_m \quad (42)$$

This value is seen to be twice as large as that computed in Equation (39). The condensate produced is thus proportional to the greenhouse volume.

Case 3

Repeat the analysis of Case 1 with the same conditions except for relative humidity values of: 55%, 65%, 85%, 95%.

Analysis

In the analysis of Case 1, the relative humidity enters the computation in Equation (32) where the daytime partial pressure of the water vapor P_{H_2O} is calculated. This partial pressure in turn is used to calculate the daytime mass of water vapor of the mixture in Equation (33).

By repeating the calculations of Equation (32) for the various relative humidity values, and then by calculating the daytime water vapor mass as in Equation (1.4), we have the following results.

$\phi(\%)$:	55	65	75	85	95
p_{H_2O} (psi):	0.279	0.3297	0.3805	0.4312	0.4819
$m_{H_2O} _{day}$ (psi)	0.868	1.026	1.184	1.342	1.499

(43)

where we have included the results of Case 1.

In each of these instances, the mass of water vapor in the nighttime air/water vapor mixture is: 0.5865 lb_m. By subtracting this nighttime water vapor mass from the masses at the various humidities, we obtain the condensates:

$\phi(\%)$	55	65	75	85	95
condensate (lb _m)	0.282	0.439	0.598	0.755	0.913

(44)

Case 4

Repeat the analysis of Case 1 with the same conditions except for daytime temperatures of: 90°F, 100°F, 110°F, 120°F, 130°F, 140°F, 150°F, 160°F and relative humidity of 85%.

Analysis

In reviewing the analysis of Case 1 we see that the temperature and humidity affect the partial pressure of the water vapor. If the temperature is 90°F we have:

$$90^\circ\text{F} = 549.67^\circ\text{R} = T_{day} \quad (45)$$

From the saturation temperature steam tables (Keenan et al. 1969a) we have

$$p_g = 0.6988 \text{ psi} \quad (46)$$

Therefore, from Equation (27) with the humidity at 85% we find the partial pressure from the water vapor to be:

$$p_{H_2O} = \phi p_g = (0.85)(0.6988) = 0.5940 \text{ psi} \quad (47)$$

As before, the ideal gas law [Equation (10)] provides the mass of the water vapor in the daytime air/water-vapor mixture as:

$$m_{H_2O} = \frac{p_{H_2O} V}{R_{H_2O} T} = \frac{(0.5940)(1000)(144)}{(85.76)(549.67)} = 1.814 \text{ lb}_m \quad (48)$$

The nighttime conditions are the same as in Case 1 with the mass of water vapor in the air/water-vapor mixture being:

$$m_{H_2O} |_{night} = 0.5865 \text{ lb}_m \quad (49)$$

By subtracting between the results of Equations (48) and (49) we obtain the condensate as:

$$\text{condensate: } 1.814 - 0.5865 = 1.228 \text{ lb}_m \quad (50)$$

By the same analysis we find the condensate at 100°F, 110°F, 120°F, 130°F, 140°F, 150°F, and 160°F to be:

T(°F)	100	110	120	130	140	150	160
Condensate (lb _m)	1.8	2.6	3.5	4.8	6.4	8.2	10.34

(51)

Case 5

Example using SI units

Given; Greenhouse volume: $V = 30\text{m}^3$; pressure: $p = 101.36 \text{ kPa}$; relative humidity: $\phi = 85\%$; temperature: $T = 30^\circ\text{C}$.

Find

The water condensed when the air/water-vapor mixture is cooled to 10°C.

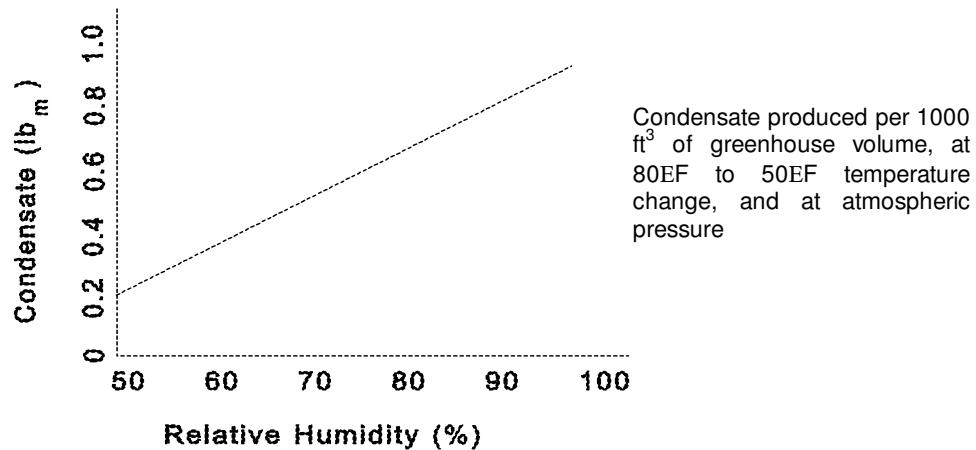


Figure 5. Effect of greenhouse volume increase upon condensate produced.

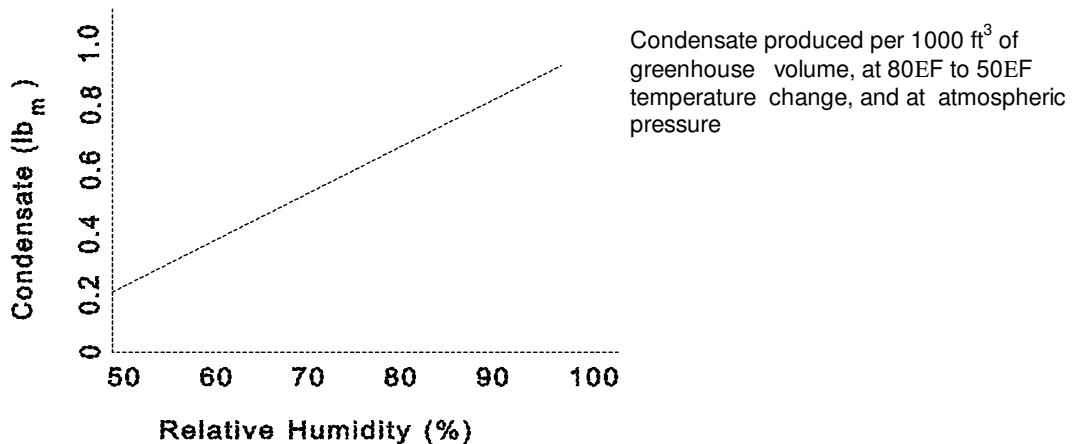


Figure 6. Effect of daytime greenhouse relative humidity increase upon condensate produced.

Analysis

During the day we have:

$$30^{\circ}\text{C} = 303.15 \text{ K} \quad (52)$$

From the steam tables (Keenan et al. 1969b), if the water vapor is in the saturation state, the saturation temperature tables at 30°C yield:

$$p_g = 4.301 \text{ kPa} \quad (53)$$

From Equation (27) the partial pressure of the water vapor is then:

$$p_{\text{H}_2\text{O}} = \phi p_g = (0.85)(4.301) = 3.656 \text{ kPa} \quad (54)$$

Using the ideal gas law [Equation (10)] we see that the mass of water vapor in the daytime mixture is then:

$$m_{\text{H}_2\text{O}} \Big|_{\text{day}} = \frac{p_{\text{H}_2\text{O}} V}{R_{\text{H}_2\text{O}} T} = \frac{(3.656)(30)}{(0.462)(303.15)} = 0.783 \text{ kg} \quad (55)$$

During the night we have:

$$10^{\circ}\text{C} = 283.15 \text{ K} \quad (56)$$

From the saturation temperature tables (Keenan et al., 1969b) at 10°C p_g is:

$$p_g = 1.244 \text{ kPa} \quad (57)$$

It is readily seen that 10°C is well below the dew point temperature. Therefore, the nighttime relative humidity ϕ is 100%. Hence, the partial pressure of the water vapor is:

$$p_{\text{H}_2\text{O}} = \phi p_g = (1.0)(1.244) = 1.244 \text{ kPa} \quad (58)$$

Using the ideal gas law (Equation (10)) we see that the mass of water vapor $m_{\text{H}_2\text{O}}$ in the night time mixture is:

$$m_{\text{H}_2\text{O}} \Big|_{\text{night}} = \frac{(1.244)(30)}{(0.462)(283.15)} = 0.285 \text{ kg} \quad (59)$$

Finally, by subtracting the water vapor masses between the daytime and the nighttime (Equations (55) and (59)), we obtain the mass of the condensate as:

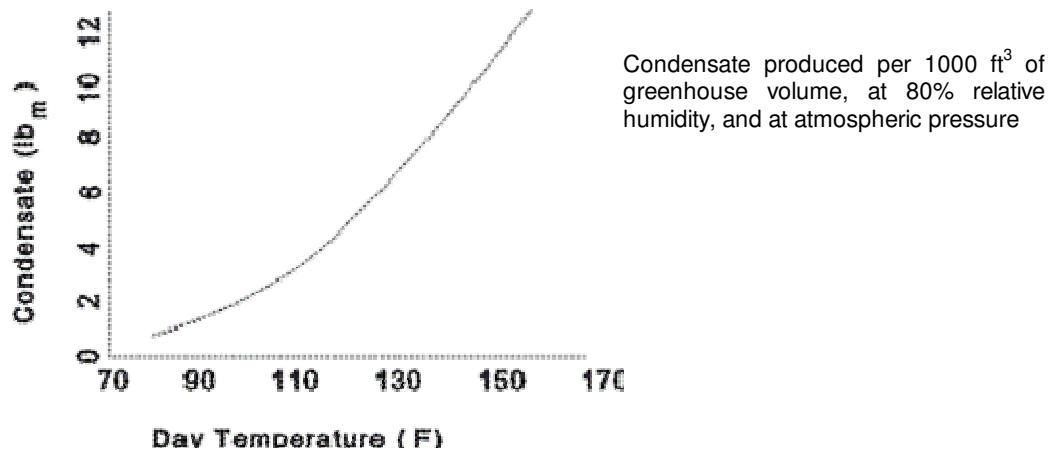


Figure 7. Effect of daytime greenhouse temperature increase upon condensate produced.

$$\text{condensate} = 0.783 - 0.285 = 0.498 \text{ kg} \quad (60)$$

RESULTS AND DISCUSSION

Figures 5, 6, and 7 provide a pictorial summary of the results of these numerical cases. The figures show that there are measurable increases in condensate produced depending upon the greenhouse volume and its temperature and humidity.

First, we see that the condensate produced is directly proportional to the greenhouse volume. This means that there is no theoretical limit to the amount of condensate that may be produced -- although there are, of course, practical limits.

Next, Figure 6 shows that the condensate is also proportional to the relative humidity of the air/water-vapor mixture of the greenhouse.

Finally, in Figure 7 we see that the condensate increases exponentially with the temperature of the daytime air/water-vapor mixture in the greenhouse.

Irrigation Feasibility

Annual rainfall is a common measure of suitability for agricultural development. Interestingly we can express the foregoing computation results in terms of equivalent annual rainfall.

To see this, recall that at standard atmospheric pressure (14.7 psi or 101 kPa) the density of liquid water is approximately: 62.4 lb_m/ft³ or 103 kg/m³. By dividing the mass of water produced by the density we immediately obtain the volume of water produced. Then by dividing by a given area of interest, we can obtain an equivalent height of

daily rainfall and by multiplication by 365.25 we have the equivalent annual rainfall.

Suppose an area A_{irr} to be irrigated is a multiple f of the base area A of the greenhouse. That is,

$$A_{\text{irr}} = fA \quad (61)$$

Suppose further that the average height of the greenhouse is: h_{avg} so that the greenhouse volume V is:

$$V = h_{\text{avg}} A \quad (62)$$

Finally, knowing that the mass m of daily water produced is directly proportional to the volume V of the greenhouse, we see that the equivalent height h_{an} of annual rainfall may be expressed as:

$$h_{\text{an}} = 365.25 h_{\text{daily}} = 365.25 m / \gamma A_{\text{irr}}$$

$$= 365.25 m / \gamma f A = 365.25 \kappa V / \gamma f A$$

or

$$h_{\text{an}} = 365.25 (\kappa / \gamma f) h_{\text{avg}} \quad (63)$$

where K is the proportionality factor for mass m of water produced per greenhouse volume V, and where the last equality follows from Equation (62).

Equation (63) shows that for a given greenhouse environment (daytime/nighttime temperature and humidity) the equivalent annual rainfall is proportional to the average height of the greenhouse.

To illustrate the use of Equation (63) consider again Case 1 where the daytime temperature is 80° F with 75% relative humidity and the nighttime temperature is 50°. For a 1000 ft³ greenhouse the daily condensate is 0.5975 lb_m (Equation (39)). For an average greenhouse height of say 10 ft the equivalent annual rainfall is:

$$h_{\text{an}} = (0.5975 / 62.4)(10 / 1000)(12)(365.25) = 0.42 \text{ in} \quad (64)$$

By comparison, a typical annual rainfall in the fertile Midwest

United States is about 40 in (e.g., Wichita, KS (Hammond World Atlas 1985). The water production from the small cubical greenhouse of Case 1 is thus insignificant.

Suppose, however, that we have a large structure say 1000 ft long, 200 ft wide ($2 \times 10^5 \text{ ft}^2$ base) with an average height of 100 ft ($2 \times 10^7 \text{ ft}^3$). Suppose further that the daytime interior temperature is now 150°F with 85% relative humidity and a nighttime temperature of 50°F . Case 5 shows that for a 10^3 ft^3 greenhouse the daily condensate is 8.272 lb_m (Equation (51)). Then for the $2 \times 10^7 \text{ ft}^3$ structure the equivalent annual rainfall produced is:

$$h_{\text{an}} = (8.272 / 62.4) (2 \times 10^7 / 10^3) (/ 2 \times 10^5) (12) (365.25) = 58 \text{ in} \quad (65)$$

Observe the dramatic increase with the large structural scale and with the high interior daytime temperature.

DISCUSSION

These results show that we can proportionately increase the water production by simply increasing the greenhouse volume, and that we can exponentially increase production by increasing the interior daytime temperature. Moreover, with the temperature increases there will be commensurate increases in humidity since the liquid lagoon water will also become warmer, increasing the evaporation.

Recent and continuing advances in solar energy technology are producing materials which can greatly enhance interior greenhouse heating.

If our objective is to produce and collect water, as opposed to direct irrigation, we can channel the dripping water inside the greenhouse to an exterior collecting container. For example, we can install longitudinally inclined, and gravity actuated, louvers to channel the accumulating

water to a tank on the shore. That is, louvers with a slightly curved (shallow) cross section, would be vertical in the daytime but then passively become horizontal as the dew falls in the nighttime.

Our envisioned structures will work best in stable climates with consistent day/night temperature changes. There are many such locations in tropical latitudes.

A distinct benefit of the structures is that it is a completely "green" technology. It can be viewed as an under roof cloud/rain producer.

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