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Review

Computerized analysis and determination of the theoretical dispersion curve of layered media under water for offshore geotechnical and geological explorations

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In offshore geotechnical and geological engineering, the determination of in-situ elastic and geometric properties of ground layers under water is of great importance. The surface wave method is an in-situ nondestructive testing technique used for engineered geotechnical explorations on land and under water. In order to apply the surface wave method under water, a computer program named SWAT has been developed during this research work, by employing the theory of wave propagation in layered media under water, and applied to construct the theoretical dispersion curve of the layered structure model. This theoretical dispersion curve is then used to make the required comparisons with the experimental dispersion curve obtained in the field for the purpose of determining the real site profile. To evaluate the applicability and reliability of the developed program, an artificial layered model as well as two examples of real sites has been studied and the results obtained have been compared with the available actual data. The results show very good match indicating that the developed program is working well and can be used for profiling the offshore sites under investigation.

Keywords: Dispersion curve, nondestructive testing, sea floor profiling, layered media, Scholte wave.

INTRODUCTION

The surface wave method is a new nondestructive testing technique for in-situ evaluation of elastic moduli and layers thicknesses of layered systems such as natural deposits and soil sites on land and under water. The method is based on the dispersive characteristics of surface (Rayleigh or Scholte) waves in layered media, i.e., waves of different wavelengths propagate with different velocities. The methodology of the test is simple and has the potential to be fully automated.

The surface wave testing procedure can be divided into three phases: (1) data collection; (2) evaluation of the experimental dispersion curve; and (3) inversion of the dispersion curve. Elastic waves are generated by an impact source on the surface of the ground, detected by at least a pair of receivers, and recorded by a transient recorder. The dispersion curve for a single receiver spacing is obtained from the relationships for phase velocity and wavelength for an arbitrary frequency component. The test is repeated for several receiver spacings to cover a broad range of wavelengths and in two directions to minimize the effects of dipping layers and the internal phase of the instrumentation. The dispersion curves for all receiver spacings and two directions are fully filtered and statically combined to derive an average dispersion curve for the system.

The objective of the surface wave test is to obtain the experimental dispersion curve that describes the relationship between the velocity of wave propagation (phase velocity) and frequency. Figure 1 shows a schematic diagram of the surface wave testing setup.

For the purpose of offshore explorations, the surface wave method consists of exciting Scholte wave energy

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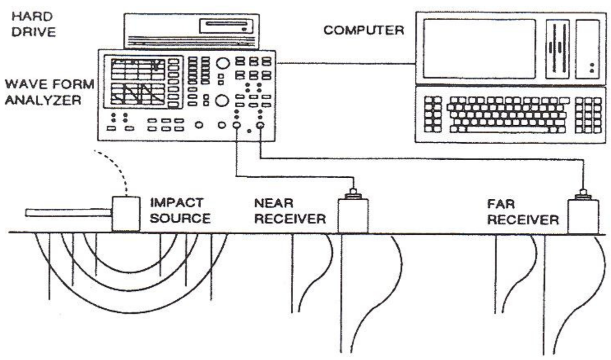


Figure 1. Schematic diagram of surface wave test setup

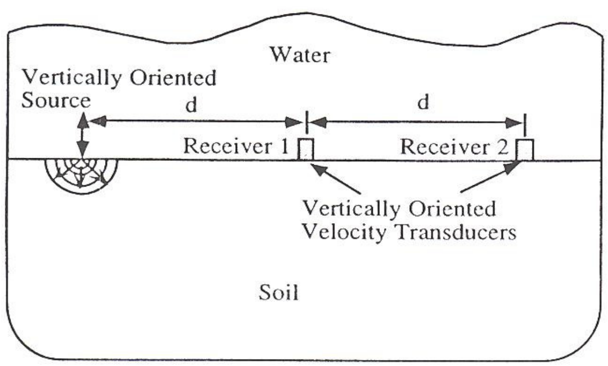


Figure 2. Field arrangement of setup under water

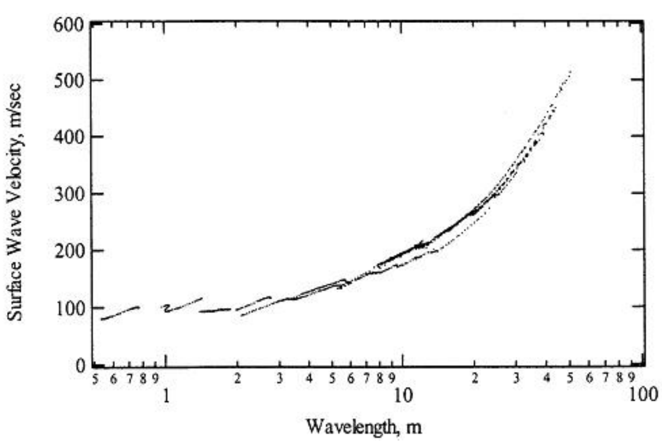


Figure 3. Typical results of surface wave field measurements

on the seafloor and measuring the propagation of this energy past two or more receivers arranged in a linear array. Scholte waves, which are also called water-soil interface waves, propagate along the water–soil interface with its energy in the two media (Wright et al., 1994). Scholte waves are generated by applying a dynamic vertical load to the seafloor surface. Vertically sensitive receivers, located on the seafloor surface, are used to sense the passage of wave energy. Figure 2 shows a typical experimental arrangement of the method under water.

The time signals recorded at each receiver are transformed into the frequency domain through a FFT algorithm, and the phase difference between the receivers, as a function of frequency, is determined. The phase angle of each digitized motion for Scholte waves is given by:

$$\phi_i = \tan^{-1}\left(\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right) \tag{1}$$

in which:
Z = complex digitized motion in frequency domain
Im(z) = imaginary part of z
Re(z) = real part of z

The phase lag of the motions between the two observed points of the receivers is then given by:

$$\Delta\phi_i = \phi_{1i} - \phi_{2i} \tag{2}$$

in which the subscripts 1 and 2 refer to receiver number. The corresponding wave length, L_i, can be calculated from:

$$L_i = \frac{2\pi}{\Delta\phi_i} d \tag{3}$$

in which *d* is the distance between receivers.
The phase velocity of the Scholte wave is computed from:

$$V_{Sc} = f\,L \tag{4}$$

where *f* is frequency and *L* is corresponding wavelength. The resulting phase velocities are plotted against wavelength (or frequency) to create a dispersion curve. A typical result of field measurements in the form of a dispersion curve is shown in Figure 3.

One of the key characteristics of Scholte waves is that different wavelengths stress different depths into the seafloor. Shorter wavelengths (higher frequencies) stress shallower depths while longer wavelengths (lower frequencies) stress deeper depths. Therefore, the trend exhibited in the measured dispersion curve with increasing wavelength reflects the stiffness properties of the material with increasing depth.

To determine the shear stiffness profile of the material, an iterative forward modeling or inversion scheme must be employed in which a theoretical dispersion curve for an assumed profile is compared to the measured dispersion curve. The profile is changed until a match between the theoretical and experimental dispersion curves is achieved.

The theoretical dispersion curve for the under water measurements will be different from the dispersion curve generated for the same profile on land due to the presence of the overlying water layer. On land, the air/soil interface closely approximates the theory of surface wave propagation at a solid/vacuum boundary.

The propagation velocity of this wave, called a Rayleigh wave, depends on the shear wave velocity and Poisson's ratio of the soil. Under water, the soil/water interface gives rise to different interface wave termed a Scholte wave. The Scholte wave velocity depends on additional factors including the depth of the water, the stiffness of the seafloor relative to the bulk modulus of the water, and the mass density of the seafloor relative to the mass density of the water. In order to develop and apply the method under water, theoretical relationships of seismic waves propagated in soil and surface waves propagated at soil-water interface have been reviewed and presented.

Stiffness matrix for a soil–water system

The stiffness matrix approach is in this study used to relate the forces at the interfaces between layers directly to the displacements at the same location. By employing this method, a global stiffness matrix of the complete layered system can be assembled. The global load vectors correspond to external forces at the interfaces of the layered system. The characteristic equation can be formulated from the stiffness matrix approach to determine normal modes of vibration of the layered system. Theoretical dispersion curves for surface waves propagating across a layered system can then be determined from the normal modes (roots). Gravity effects due to contrasting densities across boundaries at the air-water, soil-water, and soil-soil interfaces, sometimes termed "counter buoyancy" effects, are also considered in the formulation. A complete explanation of the formulation can be found in Lee, et al. (1997).

Stiffness matrix for water

The dispersion curve for a layered system overlaid by water is computed from the global stiffness matrix for the coupled layered soil-water system. The water is assumed to be inviscid and compressible, and the water motion is assumed to be irrotational. The waves traveling along the air-water interface are assumed to be of small amplitude following linear wave theory. The soil in the layered system is assumed to be elastic and its properties are characterized by shear and compression wave velocities, a mass density, a Poisson's ratio, and a layer thickness.

The governing equation for water is derived from the equation of motion and Hook's law. Displacements are also assumed to be small following linear wave theory. In general, the equations of motion for plane wave propagation can be derived from dynamic equilibrium in the two given directions. The stiffness matrix for water layer can be expressed as (Haskell, 1953):

$$K = \frac{\omega^2 \rho_w}{\alpha} \begin{bmatrix} \dfrac{e^{\alpha d} + e^{-\alpha d}}{e^{\alpha d} - e^{-\alpha d}} - \dfrac{kg}{\omega^2} & 0 & -\dfrac{2}{e^{\alpha d} - e^{-\alpha d}} \\ 0 & 0 & 0 \\ -\dfrac{2}{e^{\alpha d} - e^{-\alpha d}} & 0 & \dfrac{e^{\alpha d} + e^{-\alpha d}}{e^{\alpha d} - e^{-\alpha d}} \end{bmatrix} \tag{5}$$

which is a 3 × 3 symmetric matrix satisfying *K U* = *T* relationship, where *U* and *T* are vectors representing the displacements and stresses, respectively, at the top and bottom of the water layer. In the stiffness matrix above, *g* is the acceleration due to gravity, ρ_w is the mass density of the water, *d* is the water depth, ω is the angular frequency, *e* is the base of the natural logarithm, and α is given by:

$$\alpha = \sqrt{k^2 - \frac{\omega^2}{V_w^2}} \tag{6}$$

where V_w is the compression wave velocity of water and *k* is the wave number.

Stiffness matrix for a soil layer

The stiffness matrix for a soil layer is obtained by relating the horizontal and vertical tractions, to the horizontal and vertical displacements at the top and bottom of the layer. Kausel and Roesset (1981) showed that in plane strain conditions, the dynamic stiffness matrix for a soil layer is a symmetric 4 × 4 matrix consisting of four 2 × 2 submatrices satisfying the following relationship:

$$K = 2kG \begin{bmatrix} [k_{11}]_{2*2} & [k_{12}]_{2*2} \\ [k_{21}]_{2*2} & [k_{22}]_{2*2} \end{bmatrix}_{4*4} \tag{7}$$

The submatrices k_{11}, k_{12}, k_{21} and k_{22} are expressed as:

$$k_{11} = \frac{1-s^2}{2D} \begin{Bmatrix} \dfrac{1}{s}(C^r S^s - rsC^s S^r) & -(1 - C^r C^s + rsS^r S^s) \\ -(1 - C^r C^s + rsS^r S^s) & \dfrac{1}{r}(C^s S^r - rsC^r S^s) \end{Bmatrix} - \frac{1+S^2}{2} \begin{Bmatrix} 0 & 1 \\ 1 & 0 \end{Bmatrix} \tag{8}$$

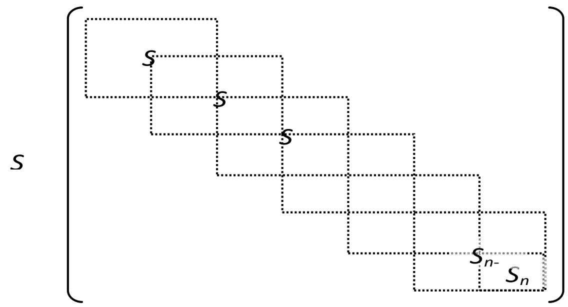


Figure 4. Global stiffness matrix for a layered system

$$k_{12}=\frac{1-s^2}{2D}\begin{Bmatrix}\frac{1}{s}(rsS^r-S^s) & -(C^r-C^s)\\ C^r-C^s & \frac{1}{r}(rsS^s-S^r)\end{Bmatrix} \quad (9)$$

$$k_{21}=\frac{1-s^2}{2D}\begin{Bmatrix}\frac{1}{s}(rsS^r-S^s) & (C^r-C^s)\\ -(C^r-C^s) & \frac{1}{r}(rsS^s-S^r)\end{Bmatrix} \quad (10)$$

$$k_{22}=\frac{1-s^2}{2D}\begin{Bmatrix}\frac{1}{s}(C^rS^s-rsC^sS^r) & (1-C^rC^s+rsS^rS^s)\\ (1-C^rC^s+rsS^rS^s) & \frac{1}{r}(C^sS^r-rsC^rS^s)\end{Bmatrix}$$

$$+\frac{1+S^2}{2}\begin{Bmatrix}0 & 1\\ 1 & 0\end{Bmatrix} \quad (11)$$

and $$D=2(1-C^rC^s)+(\frac{1}{rs}+rs) \quad (12)$$

where, $r=\sqrt{1-(\omega/kC_p)^2}$, $s=\sqrt{1-(\omega/kC_s)^2}$, C^r = *Cosh krh*, S^r = *Sinh krh*, C^s = *Cosh ksh*, S^s = *Sinh ksh*, h is layer thickness and G is the shear modulus of the layer.

The soil layer stiffness matrix is a function of the frequency of the excitation (ω), the wave number (k), the soil properties and the thickness of the layer (h).

Stiffness matrix for a half space

The half space stiffness matrix is also obtained by relating the horizontal and vertical tractions to the horizontal and vertical displacements at the soil surface. Kausel and Roesset (1981) showed a 2 × 2 stiffness matrix for a half space as:

$$K=2kG\left[\frac{1-s^2}{2(1-rs)}\begin{Bmatrix}r & 1\\ 1 & s\end{Bmatrix}-\begin{Bmatrix}0 & 1\\ 1 & 0\end{Bmatrix}\right] \quad (13)$$

Vertical displacements of the soil at the seafloor can result in counterbalancing forces due to density contrast between soil and water. This effect is referred to as the "counter buoyancy", as mentioned before, and can be taken into account in the theoretical solution by introducing a restoring force proportional to the buoyant unit weight of the soil and the vertical displacement of the midline.

Characteristic equation of the system

Stiffness matrices presented above for the water and a soil layer are used to construct the global stiffness matrix for a soil-water system. The global stiffness matrix is constructed by assembling the stiffness matrices of different layers. The scheme is shown in Figure 4. The matrix is formulated considering the continuity of displacements and force equilibrium at the top and bottom surfaces of each layer and adjacent layers.

The global stiffness matrix for a system consisting of n soil layers, with the deepest layer extending to infinity (half space), and overlaid by water is a $(2n+3) \times (2n+3)$ symmetric matrix satisfying the following relationship:

$K\,U = T$ (14)

where T and U represent vectors of external stresses (tractions) and displacements, respectively, acting at the interfaces. In solving for phase velocities corresponding to plane wave propagation in a system with no external stresses (tractions), the right hand side of above equation becomes zero ($K\,U = 0$).

The nontrivial solution to this equation is obtained by setting the determinant of the global stiffness matrix equal to zero.

$det\,[K] = 0$ (15)

This equation is the system characteristic equation relating the phase velocities to the wavelengths through sets of elastic parameters for the soil layers and water. The roots of this equation represent the velocity of possible propagating waves in the system.

An iterative procedure is employed to obtain the solution. This procedure involves assuming a trial velocity and computing the corresponding determinant of the global stiffness matrix for the system. The value for the first trial velocity is calculated using linear wave theory for a rigid base. New trial velocities are obtained by adding a velocity increment sequentially to the previous trial velocity. The velocity increment is computed by dividing the difference between the maximum shear wave velocity of the materials present in the system and the first trial velocity into a large number of increments. The existence of a root is identified when the determinants of two successive trial velocities display opposite sign. The convergence of the solution for phase velocity is based on the ratio of the

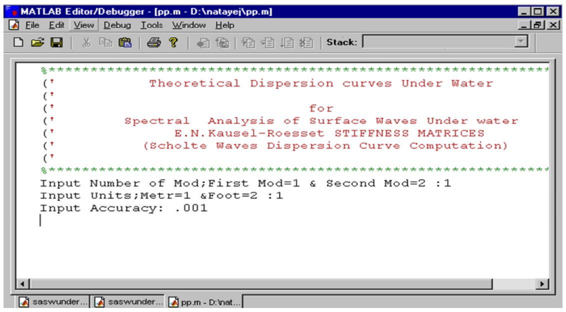
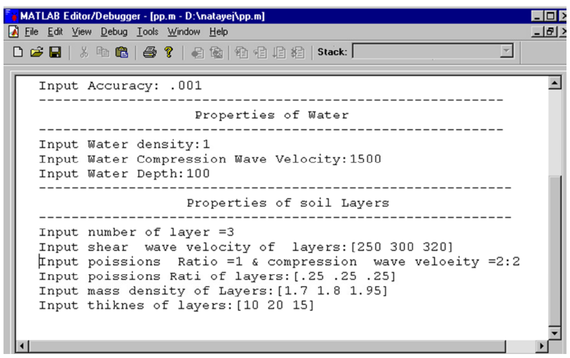


Figure 5.



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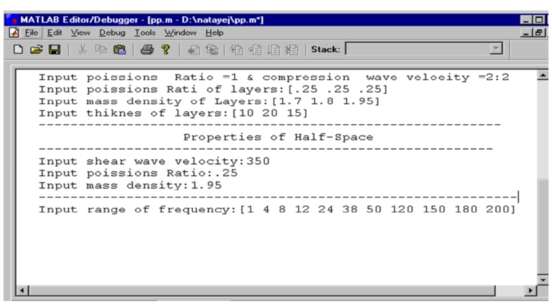

Figure 5. Three stages of data input view of the program in the *MATLAB* environment

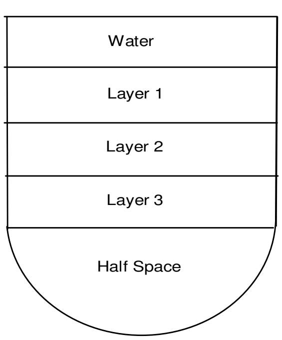

Figure 6. Layered water-soil model

change in phase velocity to the previous value of phase velocity. Convergence is considered to have been reached when this ratio is smaller than a given tolerance. A tolerance of a 10^{-6} was used in these analyses.

Computer program

A computer program named *SWAT* was written using *MATLAB* software to implement the solutions for a layered system. The program employs the iteration procedure described earlier to obtain a solution consisting of the phase velocities and displacements of the system interfaces.

The required data for the implementing the program are: water layer data including density, compression wave velocity; soil layers data including number of layers, densities, shear wave velocities, Poisson's ratios, thicknesses; and half space properties including density, shear wave velocity and Poisson's ratio. The range of frequencies of interest is also necessary to be specified. After entering the data, the program starts to construct the stiffness matrix for each layer and then assembles the stiffness matrices of different layers for constructing the global stiffness matrix. At each frequency, the program determines first the maximum wave number which makes the determinant of the global stiffness matrix equal to zero and then computes the corresponding phase velocity. The final step of the program is plotting the theoretical dispersion curve of the system using the set of pair of computed data (phase velocities and corresponding frequencies). Figure 5 shows the different stages required for data input step of the program after running the program in the *MATLAB* environment.

Application to experimental results

In order to apply and investigate the applicability of the developed computer program for constructing the theoretical dispersion curve of a layered water-soil structure, an artificial four-layer half space soil system overlaid by water has been modelled and investigated. Figure 6 shows the assumed layered profile. Also shown in Table 1 are the various values required for each layer used in the construction of the corresponding dispersion curve of assumed layered model.

Figure 7 shows the theoretical dispersion curve of the system, obtained using the developed program. The plotted theoretical dispersion curves were generated in deep water conditions (water depth of 1000 m). In deep water, the effect of water depth on Scholte wave velocity is negligible, i.e. Scholte wave velocity can be considered to be independent of the wavelength

Table 1. Parameters of water-soil layers

Layer	Thickness [m]	Shear Wave Velocity [m/se	Poisson's Ratio	Density [gr/cm^3]
Water	100	*	-	1
1	10	250	0.25	1.7
2	20	300	0.25	1.8
3	15	320	0.25	1.95
Half space	-	350	0.25	1.95

* Water compression wave velocity = 1500 *m/sec*

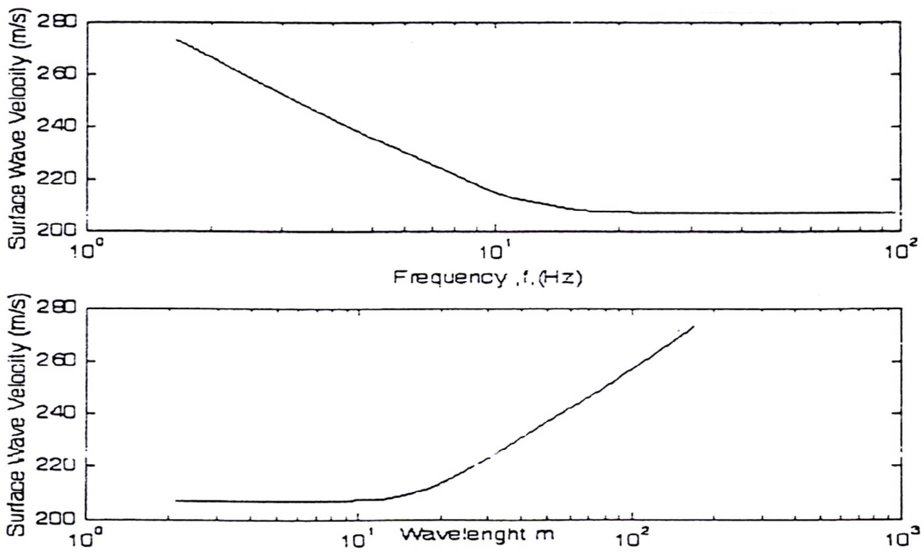

Figure 7. Theoretical dispersion curves for assumed model

[Sedighi-Manesh, et al., 1992].
The stiffness matrix for water is a 3 × 3 matrix as given below:

$$K_{water} = \begin{bmatrix} K_{11}{}^{w} & 0 & K_{13}{}^{w} \\ 0 & 0 & 0 \\ K_{31}{}^{w} & 0 & K_{33}{}^{w} \end{bmatrix} \qquad (16)$$

The global stiffness matrix for this system is a 9 × 9 symmetric matrix as indicated in relationship (17), in which $\bar{k} = (\rho_s - \rho_w)g$, is added to the appropriate diagonal term to account for counter buoyancy effects in the system due to density contrast at the soil-water interface. ρ_s is the mass density of the soil.

Besides, as an application two more cases selected from published set of data is presented and studied in this research work. The aim is to verify and also demonstrate the applicability and effectiveness of the

$$K_{ToTAL} = \begin{bmatrix} \begin{matrix} K^w_{11} & 0 & K^w_{13} \\ 0 & \multicolumn{2}{c}{\begin{bmatrix} 0 & 0 \\ 0 & K^w_{33}+\bar{k} \end{bmatrix} + \begin{bmatrix} k^1_{11} \end{bmatrix}} \\ K^w_{31} & & \end{matrix} & \begin{bmatrix} K^1_{12} \end{bmatrix} & & \\ \begin{bmatrix} K^1_{21} \end{bmatrix} & \begin{bmatrix} K^1_{22} \end{bmatrix} + \begin{bmatrix} K^2_{11} \end{bmatrix} & \begin{bmatrix} K^2_{12} \end{bmatrix} & \\ & \begin{bmatrix} K^2_{21} \end{bmatrix} & \begin{bmatrix} K^2_{22} \end{bmatrix} + \begin{bmatrix} K^3_{11} \end{bmatrix} & \begin{bmatrix} K^3_{12} \end{bmatrix} \\ & & \begin{bmatrix} K^3_{21} \end{bmatrix} & \begin{bmatrix} K^3_{22} \end{bmatrix} + \begin{bmatrix} K_{Half} \end{bmatrix} \end{bmatrix}_{9\times 9}$$

(17)

developed program and the procedure employed in the development of the program. The underwater soil layers data shown in Tables 2 and 3 are based on the experimental results given by Rosenblad (2000). The results were obtained from experiments performed

Table 2. Soil layers field measurements (Gulf of Mexico Site)

Soil Layer	1	2
Layer Thickness [ft]	1.5	HS*
Shear Wave Velocity [ft/sec]	110	700
Compression Wave Velocity [ft/sec]	5000	5000
Poisson's Ratio	0.5	0.49
Unit Weight [lb/ft^3]	90	120

*HS means Half Space

Table 3. Soil layers field measurements (Galveston Bay Site)

Soil Layer	1	2	3	4	5
Layer Thickness [ft]	2	2	2	2	HS
Shear Wave Velocity [ft/sec]	200	250	300	350	400
Compression Wave Velocity [ft/sec]	663	828	993	1158	1324
Poisson's Ratio	0.45	0.45	0.45	0.45	0.45
Unit Weight [lb/ft^3]	120	120	120	120	120

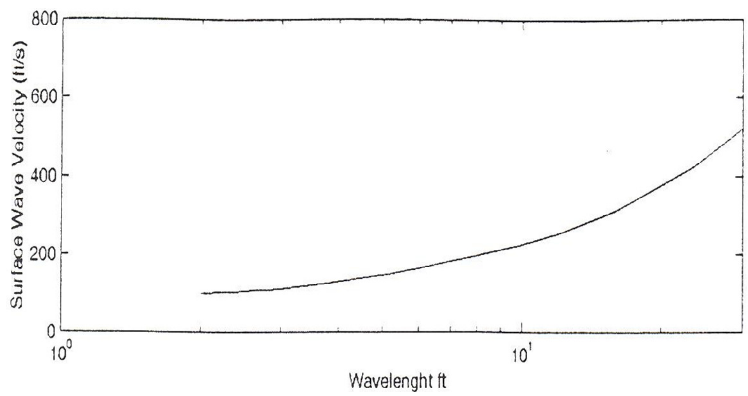

Figure 8. Golf of Mexico site dispersion curve

underwater in the Gulf of Mexico and along the East Beach area in Galveston Bay, Texas (having water depth of 40 ft and 3 ft, respectively). The results from Gulf of Mexico site were obtained using SASW and soil boring tests and the results from Galveston Bay site were measured using SASW and cross-hole tests. Also shown in the Figures 8 and 9 are the experimental dispersion curves obtained in the field for both sites.

Tables 4 and 5 compare the results between the measured phase velocities and calculated phase velocities (using the developed program) for both sites. The results have been shown for the same values of wavelengths and comparisons are being made for different values of wavelengths. As can be seen from the tables, the calculated results and the measured data are in excellent agreement. These results indicate that the program is working well and can be used effectively in the performance of surface wave method under water and offshore geotechnical explorations.

Summary and conclusion

Measurement of shear wave velocities can be an important tool for characterizing and designing land and offshore structures. The surface wave method is a nondestructive, stress-wave based test for determining shear wave velocity profiles at geotechnical sites non-intrusively. The method is based on the dispersive characteristics of surface waves in a layered system where shear wave velocities of the layers are changed with depth. Development of the method for offshore applications is in its early stages. When the test is performed underwater, the surface wave that is measured on land (Rayleigh wave) is replaced by an interface wave between the water and soil (Scholte

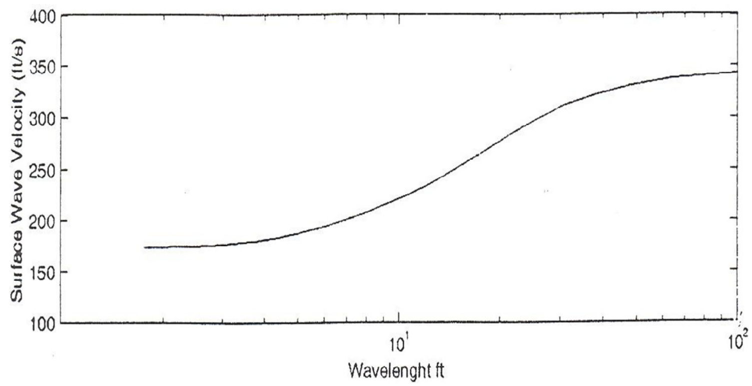

Figure 9. Galveston Bay site dispersion curve

Table 4. Comparison of measured and calculated results (Gulf of Mexico)

Wavelength [ft]]	Field Test Phase Velocity [ft/sec]]	Calculated Phase Velocity [ft/sec]
1	100	100.57
2	111	110.75
5	150	150.48
10	233	232.80
20	377	377.19

Table 5. Comparison of measured and calculated results (Galveston Bay)

Wavelength [ft]]	Field Test Phase Velocity [ft/sec]]	Calculated Phase Velocity [ft/sec]
1	180	179.60
2	180	179.60
10	226	226.35
20	282	282.18
40	330	329.84

wave). The propagation velocity of this wave depends on the stiffness of the seafloor as well as additional factors including the wavelength of the Scholte wave and water depth.

In order to develop and apply the surface wave method under water, a computer program named *SWAT* has been developed during this research work and used for constructing the theoretical dispersion curve of the layered model. This theoretical dispersion curve is then used to make the required comparisons with the experimental dispersion curve obtained in the field for the purpose of determining the real site profile.

By employing the developed program, an artificial layered model as well as two examples of real sites has been studied and the results obtained have been compared with the available actual data. The results show very good match indicating that the developed program is working well and can be used for profiling the sites under investigation.

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