



Full Length Research Paper

Comparison of the history matching and forecasting capabilities of single and multi-objective particle swarm optimisation using the punq-s3 reservoir as a case study

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Abstract

Petroleum reservoirs are characterised by various features which are indicative of the geologic processes that resulted in the formation of such rock bodies. These geologic processes are responsible for the major uncertainties surrounding oil and gas production. There is therefore the need to quantify these uncertainties to an acceptable level to allow for the certain risk evaluations on petroleum projects. The process of history matching involves creating of reservoir models which mimic the observed reservoir performance to some acceptable extent. This paper evaluates two numerical schemes by which the history matching process is optimised with the subsequent utilisation of the models obtained from this process to infer certain properties deemed to be representative of the reservoir future performance. Single objective and multi objective particle swarm optimisation algorithms are used in optimised history matching of the synthetic PUNQ-S3 reservoir with the results from the two schemes put forward to a Bayesian evaluator for forecasting. The results obtained suggest the multi objective particle optimisation scheme not only produces good quality history matches but it also converges faster. With regards to forecasting, the models obtained from both schemes did not reflect the observed well bottom hole pressures. However, the multi objective scheme provided better forecasts of the field total oil production relative to the single objective scheme with the truth case being reflected. Moreover, the uncertainty intervals created from the multi objective scheme are wider than those generated from the single objective scheme.

Keywords: Forecasting, particle, petroleum, reservoir, risk, and uncertainty.

INTRODUCTION

Hydrocarbon resources are contained in certain compartments in the earth's crust which have been subject to varying degrees of geologic history. The depositional circumstances of such oil containing reservoirs result in these geologic structures being characterised by some degree of heterogeneity. For instance, the possibility of having uniform rock permeabilities across an entire reservoir is considered to be minimal with expectation of relatively high and low permeability streaks in such formations (Mohamed et al.,

2010).

There is the need to have some sort of model to mimic the production of crude oil from these reservoirs. However, modelling of reservoir bodies is carried out with under varying degrees of uncertainty due to the geological processes which have resulted in their formation. The basic process of model generation involves an attempt to calibrate a reservoir model such that its expulsion of oil, gas and water is similar to that which has been observed. This process termed history matching is characterised by its non-unique nature. Various reservoir models may be constructed all of which represent an adequate match to observed production data. These models however are characterised by the

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different combinations of reservoir parameters.

Over time, uncertainty management practices have evolved resulting in the creating of various numerical means of quantifying the uncertainties relating to production from hydrocarbon reservoirs (Mohamed et al., 2011). Various computational means now exist through which a set of history matched reservoir models may be generated and utilized in uncertainty quantification. The fundamental idea behind these processes is the development of various sampling numerical algorithms which enable some form of optimised search for the right set of reservoir parameters which satisfy a given objective to some extent. Generally, the objective function to be satisfied involves minimisation of the deviation between the history matched or simulated reservoir models and observed production data (Mohamed et al., 2010). These history matched models are then subsequently used to infer future magnitudes of certain reservoir or well production parameters. Like the history match algorithms described above, various sampling techniques have been developed to handle this prediction process.

The performance of two forms of the particle swarm optimisation algorithm in history matching a set of synthetic data is analysed in this paper. The single objective form of the algorithm performs history match runs with emphasis on trying to minimise an objective function. Multi objective particle swarm optimization on the other hand performs this same function but with the number of objective functions to be handled not limited to one thereby permitting simultaneous investigation of a wide range of scenarios.

The results from these inverse problem estimations are then forwarded to a Bayesian evaluator where inference on the magnitude of well bottom hole pressures and comparison of the resulting Bayesian intervals from both schemes is made.

It is clearly apparent that there is a huge economic incentive to the establishment of these numerical processes. Uncertainty quantification is generally a measure of the risk oil and gas majors carry with each investment. Thus, these history matching and prediction frameworks enable the construction of various economic models in order to determine the degrees of commercial viability which may accrue to a petroleum project.

Particle Swarm Optimisation

The various optimisation techniques utilized in history matching involve some sort of numerical algorithm which samples a predetermined sample space the limits of which are subject to certain geological and engineering considerations.

The Particle Swarm Optimisation (PSO) was initially designed as a means of measuring the social behaviour of a group of birds but its application has been extended

to various fields including petroleum engineering (Hajizadeh et al., 2010). The basic thought process behind the algorithm is that birds belonging to a particular flock would generally tend to fly towards regions perceived to exhibit certain habitat characteristics at a particular point in time. Each member of the flock thus has a memory of its best historical location and the flock is often characterised by a global best position.

The Particle Swarm Optimisation process involves initiating a swarm of particles which randomly search the sample space in an attempt to obtain a sufficient solution to a particular objective function which in this case is the misfit function defined in equation 1.

$$Misfit, M = \sum_{j=1}^j \left(\frac{p^{obs} - p^{sim}}{2\sigma^2} \right)_j^2 \dots \dots \dots Equation 1$$

Where j is the number of observations; P is the parameter around which the history match is being conducted (e.g. oil flow rate, well bottom hole pressure, water cut, well gas-oil ratio etc.); σ is the standard deviation of the observed parameter, P and is often estimated by the anticipated errors in the measuring devices used in determining the magnitude of such parameters.

The Particle Swarm Optimisation algorithm like certain other numerical algorithms related to history matching may be applied to single objective and multi objective functions.

A diagrammatic illustration of the Particle Swarm Optimisation algorithm from the work of *Mohamed et al. (2010)* is shown below in Figure 1.

The overall aim of the search process is to determine that particle position within the search domain which minimises the objective function (i.e. the misfit equation) i.e.

$$\left. \begin{aligned} p_{best\ i}^{k+1} &= p_{best\ i}^k \text{ if } f(x_i^{k+1}) \geq f(p_{best\ i}^k) \\ p_{best\ i}^{k+1} &= x_i^{k+1} \text{ if } f(x_i^{k+1}) < f(p_{best\ i}^k) \end{aligned} \right\} \quad Equation 2$$

(*Mohamed et al: History Matching with Particle Swarms (2010)*).

A particle's updated position may be mathematically expressed thus:

$$x_i^{k+1} = x_i^k + v_i^{k+1} \dots \dots \dots Equation 3$$

Where X_i depicts the particle position at different times while, V_i^{k+1} is the velocity component of the update equation.

Also,

$$v_i^{k+1} = \omega v_i^k + [c_1 rand_1 (p_{best\ i}^k - x_i^k)] + [c_2 rand_2 (g_{best}^k - x_i^k)] \dots \dots Equation 4$$

Characteristics of the Particle Swarm Optimisation Algorithm

The basic equation of the algorithm is represented in equation 4 above. The meaning of the terms involved and

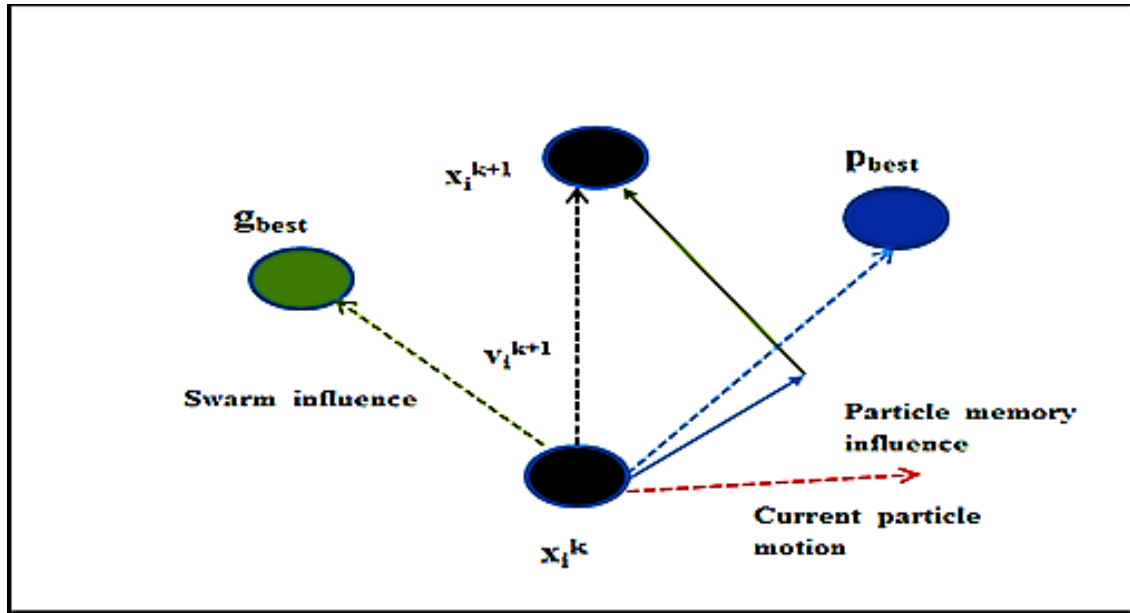


Figure 1. Illustration of Particle Swarm Optimisation Algorithm (Mohamed et al: Application of Particle Swarms for History Matching in the Brugge Reservoir)

their implications on the algorithm is summarised thus:

- ω : This depicts the inertia (i.e. the ability of a particle to move within the search domain). Large values of inertia correspond to a wider range of particle exploration and therefore a larger search region whilst smaller inertia values imply a search in a relatively narrower neighbourhood. Various suggestions have been made as to the magnitude of the inertia resulting in the development of different inertia handling schemes (Mohamed et al: History Matching with Particle Swarms (2010)). A linear decreasing inertia function for instance would imply initial relatively large scale exploration culminating in a much narrower search space whilst a linear increasing inertia function finally results in a larger particle exploration.
- C_1 : This term referred to as the cognitive component of the algorithm illustrates some form of particle interaction. It depicts the probability of a particle moving towards its perceived region of best fit according to its retained memory. $C_1 \in (1,2)$ (Mohamed et al. (2010))
- C_2 : This term referred to as the social component of the algorithm also illustrates some form of particle interaction. It depicts the probability of a particle moving towards the perceived region of best fit attained by a member of the particle swarm. $C_2 \in (1,2)$ (Mohamed et al. (2010))
- rand_1 and rand_2 : These terms are introduced to account for uncertainty with regards to the implementation of the algorithm. rand_1 and $\text{rand}_2 \in (0,1)$.

Boundaries

There is a tendency for a particle within the swarm to exit the desired search domain. Various techniques have however been developed to ensure such particles are repositioned within the search domain. Some of these include the respawn strategy, absorbing strategy, reflecting strategy and the damping strategy.

Mohamed et al. (2010) investigated the sensitivity of the particle swarm optimisation algorithm to the bouncing strategy with a linear decreasing inertia and with a constant inertia of 0.6. They observed an earlier convergence for the case with the constant inertia (0.6) most probably due to the relatively lower local exploitation associated with the constant inertia.

The basic single objective particle stream optimisation workflow may be summarised thus: (Mohamed et al: History Matching with Particle Swarms (2010))

- Initialise the particle swarm (i.e. distribute specified number of particles randomly in sample space)
- Evaluate the objective function for each particle in the swarm (i.e. equation 1)
- Compare each particle's fitness value with that corresponding to pbest. Replace pbest with the current position if the current value of the fitness function is less than that corresponding to pbest (i.e. equation 2)
- Update the global best position and fitness value of the swarm

Update the velocities and positions of each particle using equations 3 and 4

- The above processes are then repeated till some stopping criterion is reached. Stopping criteria may include number of iterations, specified misfit value or certain gradient value computations.

Multi Objective Particle Swarm Optimisation

The basic concepts which apply to single objective particle swarm optimisation schemes also apply to the multi objective scheme. However, there is an additional concept of dominance due to the need for some sort of compromise between various objectives. Dominance in this context refers to a set or sets of solutions to a multi objective problem which optimises the solution to one of such functions without having an adverse effect on the other. Mathematically, a solution X_1 is dominant over another X_2 if

$$f_i(x_1) \leq f_i(x_2); i = 1, 2, 3, \dots, k \dots \dots \dots \text{Equation 5}$$

(Mohamed et al. (2011))

Where $f_i(X)$ denotes the objective functions (misfit) to be minimised and k is the number of such functions or if X_1 is strictly better than X_2 in at least one of the objective functions. (Mohamed et al. (2011)).

Multi objective particle swarm optimisation estimates may be made on different number of objective functions. For instance one of such functions may be the well bottom hole pressure of a producer whilst the other may be the producer's Gas-Oil Ratio (GOR). In terms of the initially defined objective function (equation 1), we may write:

$$\left. \begin{aligned} \text{Misfit}, M_1 &= \sum_{j=1}^J \left(\frac{WBHP^{obs} - WBHP^{sim}_j}{2\sigma^2} \right)^2 \\ \text{Misfit}, M_2 &= \sum_{j=1}^J \left(\frac{GOR^{obs} - GOR^{sim}_j}{2\sigma^2} \right)^2 \end{aligned} \right\} \text{Equation 6}$$

Various attempts have been made to extend single objective particle swarm optimisation algorithms to multi objective schemes, however, these processes all vary based on the presumed definition of optimality. Three main techniques may be summarised thus: (Mohamed et al: *History Matching and Uncertainty Quantification: Multiobjective Particle Swarm Optimisation Approach* (2011)).

- Aggregation based methods in which the various objective functions are summed up and the optimisation process is carried out in form of these lumped sums.
- Criterion based methods in which different phases of the optimisation scheme operate on a different objective.
- Pareto dominance based methods which require the determination of a group of non-dominated solutions which are not dominated by any other feasible solutions. This method is characterised by a Pareto front, along which these non-dominated solutions are evident. The basic aims of this scheme are to obtain a diverse set of

Non-dominated solutions, decrease the distance between the Pareto front if it exists and the solutions and maximise the number of solutions along the Pareto front (i.e. the Pareto Optimal Set). (Hajizadeh et al: *Towards Multi objective History Matching: Faster Convergence and Uncertainty Quantification* (2011)). As stated earlier, various extensions of the original particle swarm optimisation have been made to account for multi objective cases. The crowding distance mechanism is discussed here.

Crowding Distance Technique

The crowding distance technique involves determination of a diverse set of optimal solutions with a good measure of distance between such solutions across the Pareto front if it exists. Selection of the swarm leader is therefore not random as in the single particle swarm optimisation algorithm but rather, the leader is selected based on the estimated crowding distance between the various members of the swarm.

Mathematically, the same update equation governs the particle position update (equation 3) however there is a change in the velocity update as shown below. (Mohamed et al. (2011))

$$v_i^{k+1} = \omega v_i^k + [c_1 \text{rand}_1 (pbest_i^k - x_i^k)] + [c_2 \text{rand}_2 (A(gbest^k) - x_i^k)] \dots \dots \text{Equation 7}$$

Crowding distance is defined as the size of the largest cuboid around a particle with the exclusion of any other particles in the swarm. (Mohamed et al. *History Matching and Uncertainty Quantification: Multiobjective Particle Swarm Optimisation Approach* (2011)).

The global swarm leader is then selected based on a descending order (the top 10%) of the crowding distances associated with the particles from an external archive. Another characteristic of the crowding distance algorithm is the addition of a mutation factor to account for the possible premature convergence of the iterations due to the particle being trapped in local optimum points. Mutation factors are often applied to the entire swarm at the commencement of the algorithm.

The general multi objective particle swarm optimisation workflow may be summarised as follows (Mohamed et al: *History Matching and Uncertainty Quantification: Multiobjective Particle Swarm Optimisation Approach* (2011)):

- Initialise the particle swarm
- Estimate crowding distance of particles in external archive and determine leader
- Repeat
 - For each particle
- Select leader
- Update velocity and position
- Consider mutation effect
- Evaluate misfit
- Update particle's best position, pbest

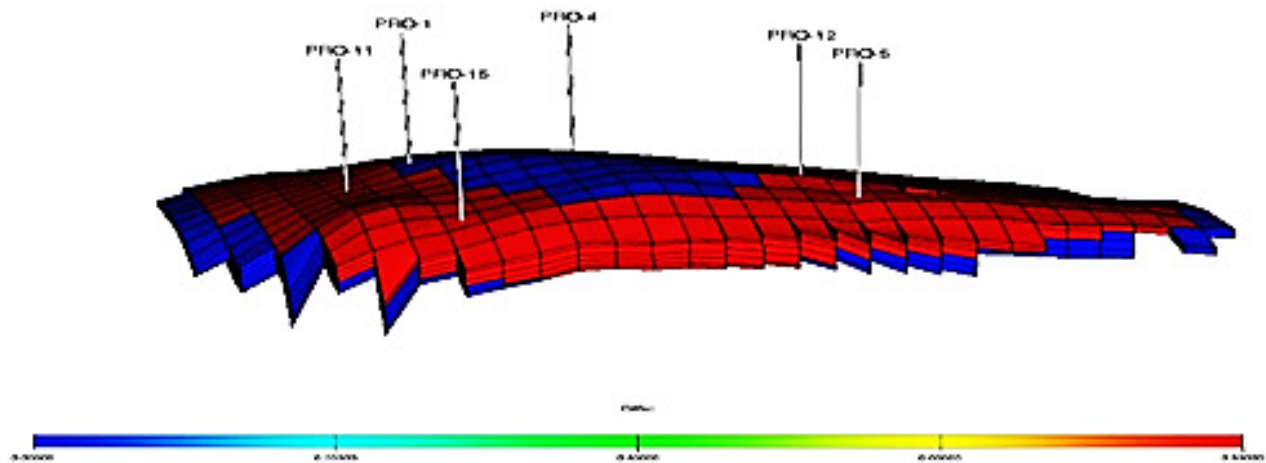


Figure 2. PUNQ Model Saturation Distribution

- Update leaders in external archive
- Perform random replacement when archive is full
- Evaluate leader and move on to next iteration till a stopping criterion is reached.

Forecasting

The history matching process as earlier discussed is not unique in terms of the solutions that emerge from it. It is possible to obtain probabilistic estimates of the reservoir's future performance based on these history matched models. Groups of such reservoir models may be termed an ensemble. The Bayesian framework for extrapolating reservoir performance is discussed briefly here. This method involves the determination of a posterior probability distribution (PPD). The Bayesian credible intervals are evaluated by integration over this multi-dimensional model space. The direct Monte Carlo methods commonly applied in this method are not applicable where the data to be handled has no defined distribution. Hence, some form of modification is required to handle these irregular distributed data models. The history matched reservoir models are a typical example of this scenario. According to *Sambridge (1999)*, two major problems arise in the determination of the PPD. These are the constraint provided by the observed data in that the history matching process is not unique and the distribution of the sampled data (reservoir models) is uncertain. Thus, it is generally accepted that the ensemble will always be inadequate.

The probability distribution density function (PPD) within a multidimensional model space, m , may be defined thus: (*Sambridge M: Geophysical inversion with a neighbourhood algorithm (1999)*).

$$P(m)|d_o = k\rho(m)L(m)|d_o \dots \dots \dots \text{Equation 8}$$

Where k is a normalising constant, $\rho(m)$ is the prior probability and $L(m)|d_o$ is the likelihood function. The likelihood function is a mathematical expression of the degree of fit between modelled and observed data and is given by the negative exponent of equation 1.

PUNQ-S3 Reservoir

The PUNQ-S3 is a synthetic reservoir model developed for various petroleum engineering studies. In this study, the geologic uncertainty in the model is examined via the already discussed numerical means and the accuracy of these discussed algorithms in uncertainty quantification is determined. It consists of five layers further subdivided into nine homogeneous units. The reservoir top is at 2430m at a dip angle of about 1.5° . There is the presence of a small gas cap in the structure which is bounded by a fault to the east and is underlain by a strong aquifer implying a good degree of pressure support (*Hajizadeh et al. (2010)*). The various production wells are as indicated on the structure (Figure 2).

The characteristics of the model are summarised thus:
Grid type: 3-D (19 x 28 x 5); only 1761 cells are active.
Grid dimensions: X = Y = 180meters.

Permeability across the reservoir is extrapolated with respect to porosity via the following mathematical expressions (*Hajizadeh et al. (2010)*).

$$\ln k_h = 0.77 + 9.03\phi \dots \dots \dots \text{Equation 8}$$

$$k_v = 3.124 + 0.306k_h \dots \dots \dots \text{Equation 9}$$

The porosity and permeability ranges utilized in the algorithm sampling process are given below in Table 1.

Table 1.Parameterization of PUNQ Reservoir Model (Hajizadeh et al. (2010))

Porosity	Horizontal Permeability	Vertical Permeability	Distribution Employed
0.15 - 0.3	133 - 3013	44 - 925	UNIFORM
0.05 - 0.15	16 - 133	8 - 44	UNIFORM
0.15 - 0.3	133 - 3013	44 - 925	UNIFORM
0.1 - 0.2	47 - 376	17 - 118	UNIFORM
0.15 - 0.3	133 - 3013	44 - 925	UNIFORM

METHODOLOGY

The analytical method utilized in this work includes performance of the history matching process using both the SOPSO and MOPSO. This was implemented on the Raven software interface which has been created for optimised history matching and uncertainty quantification.

Sensitivities were carried out initially on the basic PSO algorithm to characterise its convergence capabilities. Four alterations to the algorithm were investigated and their convergence rates determined. However, it is important to note that these sensitivities are carried out without considering the initialisation point of the algorithm in each run. Hence, general conclusions cannot be made here as to the obtained results.

The SOPSO algorithm was applied to the synthetic PUNQ field. All the desired objective functions (quantities to be history matched) were the input to the process.

For the MOPSO algorithm, only a double objective approach was considered. Two main scenarios were investigated in this work. Firstly the bottom hole pressures were grouped as one objective with the gas oil ratios across the wells serving as the second objective.

In the second scenario investigated, the wells were grouped into two separate groups with all the objective functions corresponding to wells from each group serving as the two objectives.

A 75%-25% split is used on the observed data to allow comparison of the forecasting capabilities. The results of the history match process are then forwarded to the Bayesian Neighbourhood Algorithm (NAB) for forecasting.

RESULTS AND DISCUSSION

The performance of the basic particle swarm optimisation algorithm was examined by carrying out sensitivity on the mathematical equation governing the algorithm (equation 4). The algorithm characteristics utilized in these sensitivity estimations is presented in table 2.

As observed in Figure 3 the average misfit per iteration does not follow a general reducing pattern with increasing number of iterations in three of the cases. This occurrence is due to the random nature of the algorithm initialisation probably suggesting the parameters utilized

in the standard 1 case are some type of optimum. Also, for runs of 2000 iterations performed on the four investigated set of parameters, it is observed from figure 4 that the lowest misfit within generation sets follows a general reducing trend with the flexible parameters case becoming a little better after 2000 iterations than the standard 1 set of parameters. It is important to note that due to the random nature of the swarm leader selection, the lowest misfits in each generation are a function of the location of the swarm leader within the search domain. Hence, the average misfit per iteration shown in Figure 3 may be a more realistic way of analysing these scenarios. Moreover, Figure 4 shows the same pattern as Figure 3 suggesting the parameters in the standard 1 version of the algorithm are somewhat more stable than the others. Comparative analysis of the two particle swarm optimisation schemes was also carried out. The multi objective version of the particle swarm optimisation algorithm was also examined using the standard 1 set of parameters as indicated in table 2.

The resulting best history match models across the wells for both the SOPSO and MOPSO schemes is illustrated in Figure 5 with table 3 showing the comparison of both schemes. Convergence rates of both algorithms were made as observed in Figures 6 and 7. Due to the differences in the initialisation of both algorithms the relative convergence rate of the two PSO schemes was assessed by comparing the average misfit over 3 runs each comprising of 1000 iterations. Also, the minimum objective function after 10, 50 100, 200, 500 and 1000 iterations for each of the runs was averaged and used in this comparative analysis. The results suggest that the MOPSO algorithm converges faster than the SOPSO algorithm. The results obtained here may not be generalized due to the following reasons.

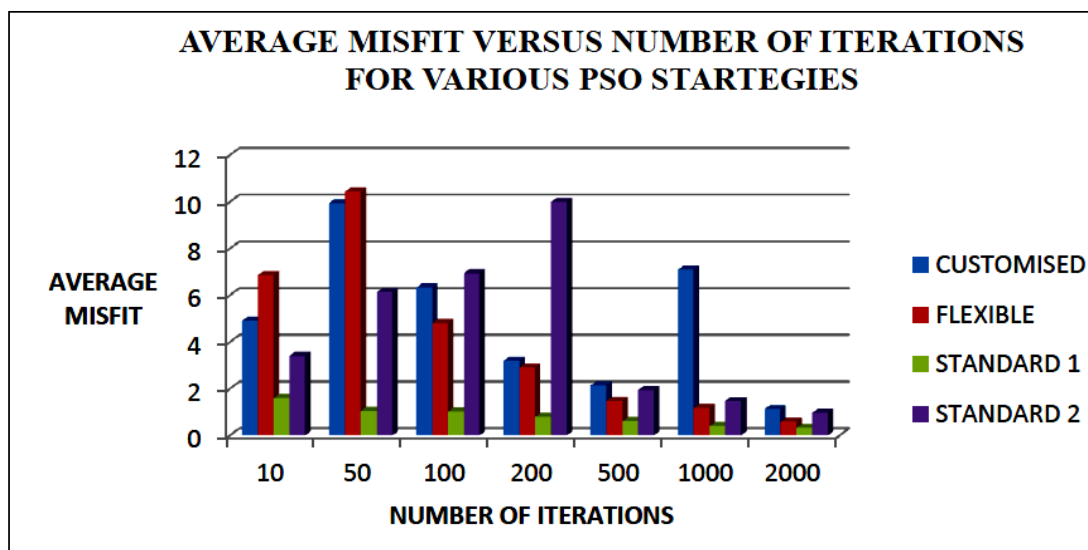
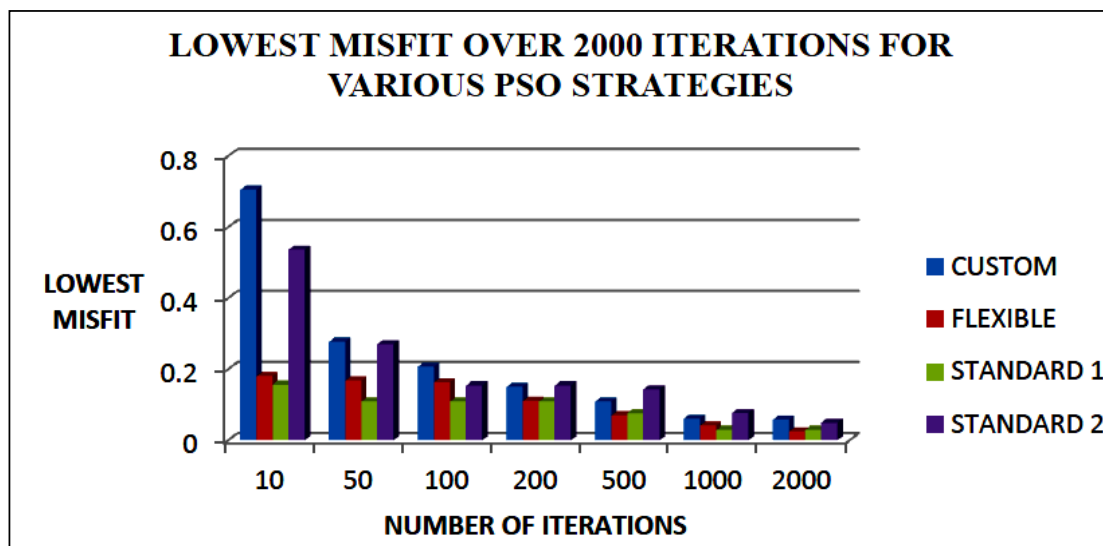
- They are specific to the PUNQ-S3 reservoir
- The initial starting point of both the SOPSO and MOPSO algorithms most likely do not coincide hence a fair assessment may not be made here.

Also, figure 8 illustrates the total objective misfits for the PSO scheme (wells grouped). These plots suggest the SOPSO scheme obtains relatively better history matches.

Also, a set of non-dominated solutions has been identified by combination of the misfits obtained for the MOPSO scheme (wells grouped). This is shown in Figure 9. The Pareto front is approximated to define the left and

Table 2. Characteristics of various PSO strategies investigated

	No of particles	C1	C2	Ω
STANDARD1	20	1.494	1.494	0.729
STANDARD2	20	1.7	1.7	0.6
FLEXIBLE	20	1.333	1.333	0.9
CUSTOMISED	20	2	2	0.4

**Figure 3.** Average misfit for different generation sets for various PSO strategies**Figure 4.** Lowest misfit for different generation sets for various PSO strategies

bottom envelope of the plots.

The complete set of 1000 history matched models was submitted to the Bayesian evaluator for determination of

the Bayesian credible intervals. The exact extent of the sampling process resulting in the generation of the intervals is uncertain.

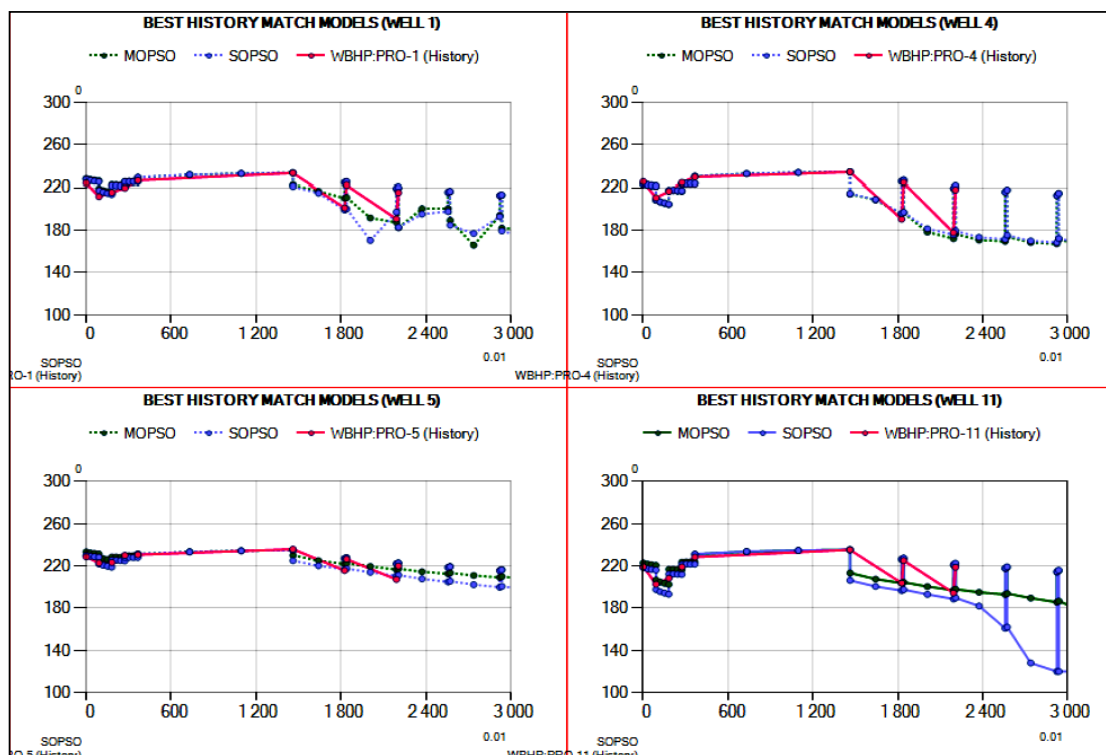


Figure 5. Best History Match models (MOPSO Scheme - Wells grouped)

Table 3. Comparison of both SOPSO and MOPSO schemes

	No. of iterations	Minimum objective function	Generational minimum (last iteration)
SOPSO	1000	0.54	0.75
MOPSO (BHPS and GORS GROUPED)	1000	0.19	1.42
MOPSO (WELLS GROUPED)	3000	0.2	1.60

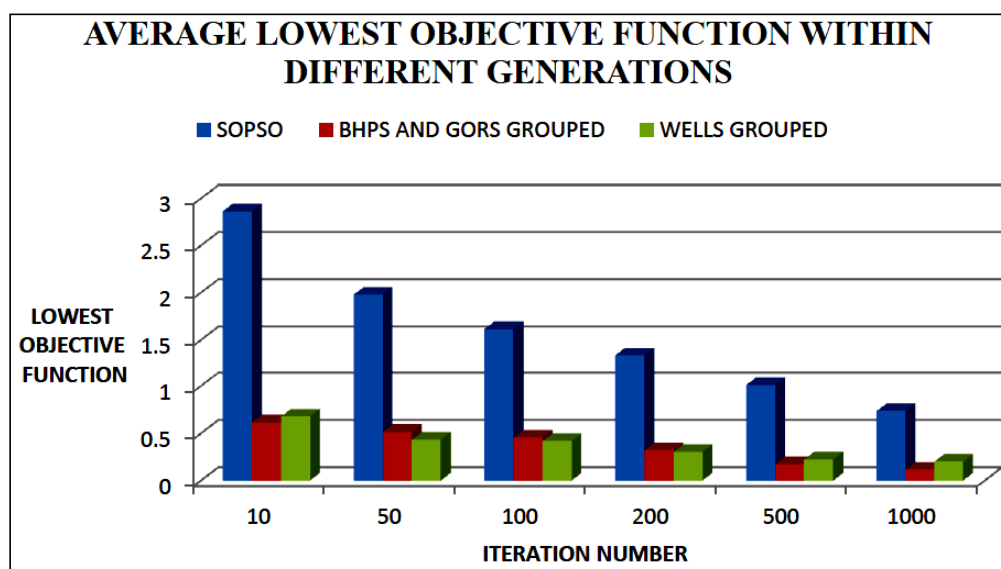


Figure 6. Average Generational minimum misfit for different generation sets

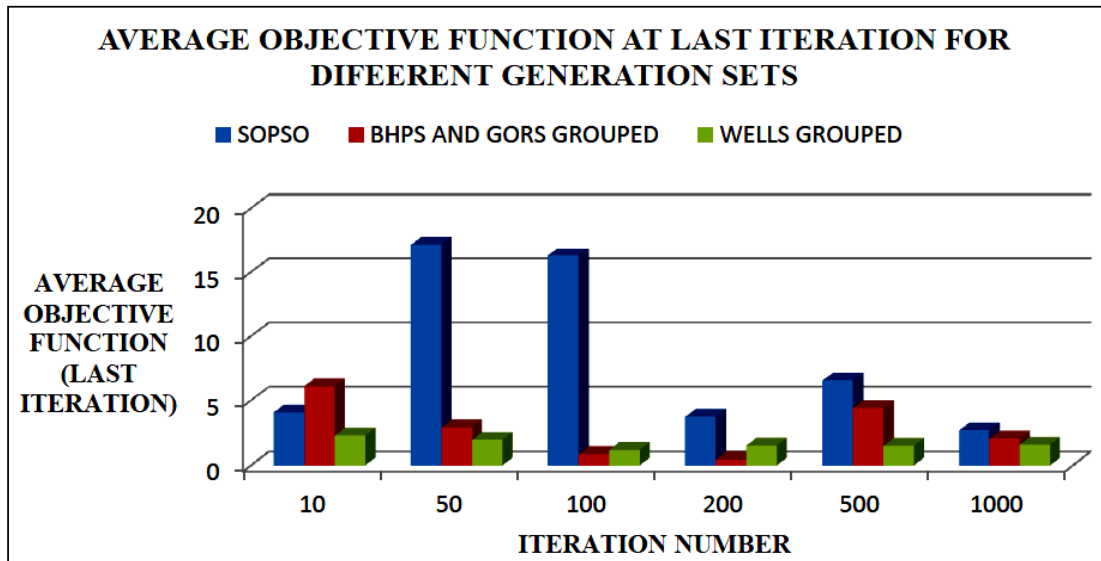


Figure 7. Average lowest objective function within different generations

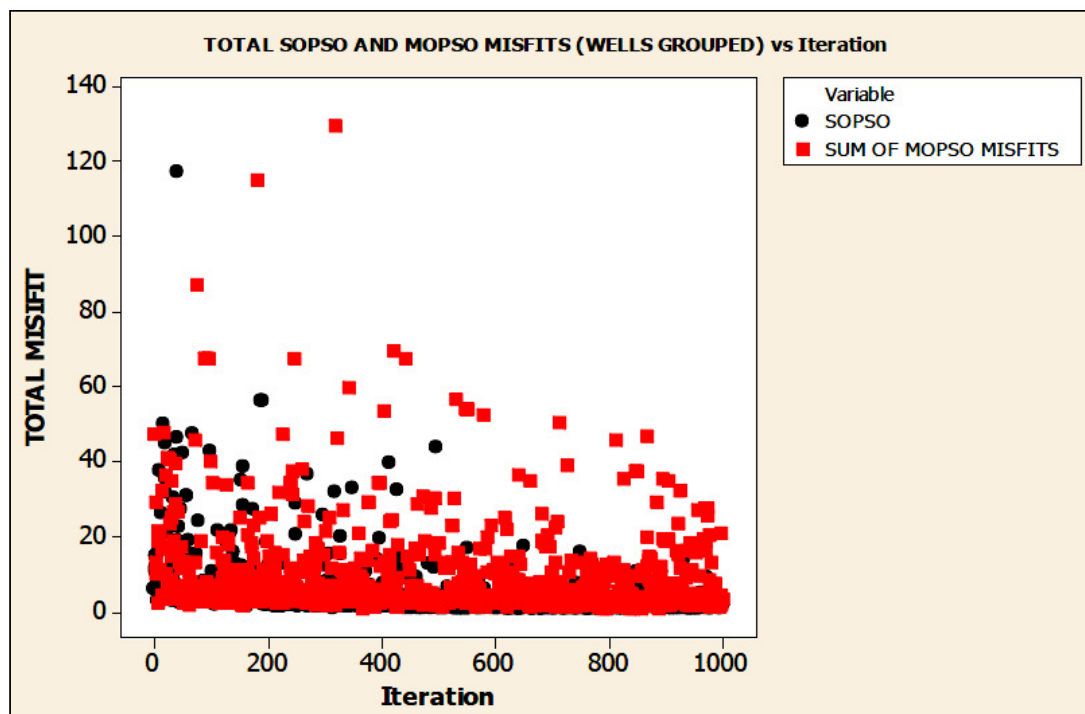


Figure 8. Total Misfits against number of iterations for both PSO schemes

In terms of total field oil recovery, the truth case cumulative oil production at 2571 days is used for comparison as shown in Figure 10. It is observed that the MOPSO scheme generally produces a wider interval range which is reasonable in terms of uncertainty quantification.

CONCLUSION

The following conclusions maybe reached considering the results obtained in this research work.

- Both of the particle swarm optimisation schemes evaluated (SOPSO and MOPSO) are important tools for

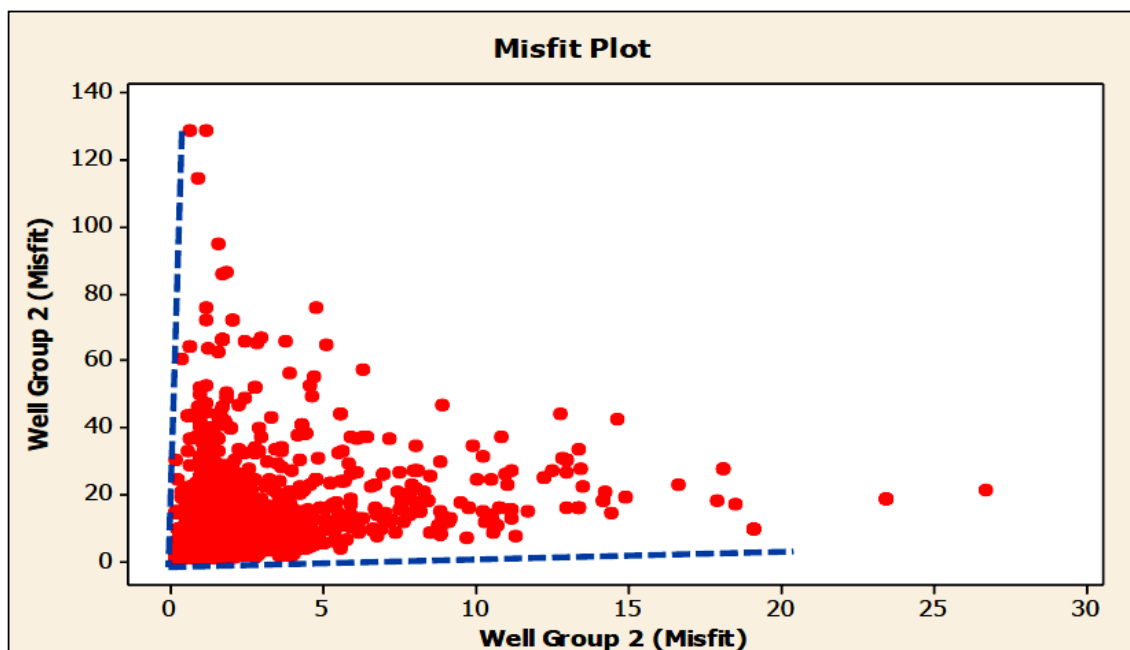


Figure 9. Misfit Plot showing approximate Pareto front (Wells grouped)

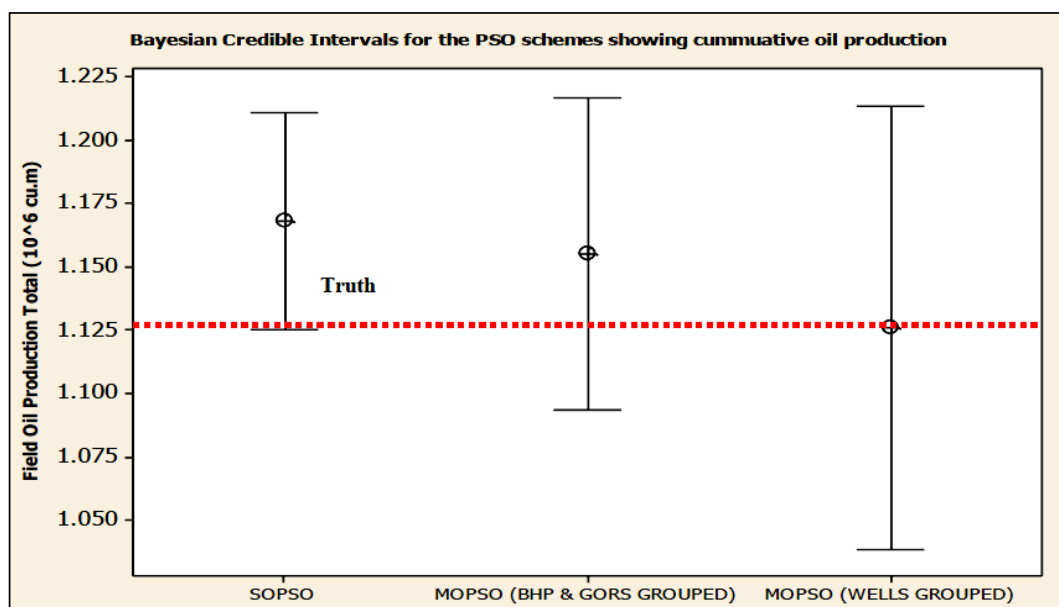


Figure 10. Bayesian credible intervals for both PSO schemes showing total oil production at 2571 days

Uncertainty quantification

- The MOPSO scheme allows multiple realisations of the reservoir performance to be evaluated as groups of objective functions.
- The MOPSO scheme converges faster than the SOPSO scheme implying that fewer simulations may be needed to obtain good quality history matches. However

this observation is subject to further investigation on different reservoir models and the same algorithm initialisation point within the search domain.

- The models obtained from the history match process may be used to infer certain quantities which would characterise future reservoir performance. The MOPSO scheme obtained reasonably wider uncertainty

intervals with respect to the total oil production whilst the SOPSO scheme was inadequate in providing a similar result.

▪ In terms of economic worth, the value of both schemes cannot be over emphasized. The uncertainty ranges from the Bayesian evaluator may be incorporated into various financial models thereby enabling the commercial viability of petroleum projects to be determined. Thus, the associated risk in oil and gas projects may be quantified.

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