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Full Length Research Paper

Children's difficulties with division: an intervention study

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Abstract

This intervention study aimed at improving children's understanding of division. One hundred low-income Brazilian children aged eight to 11 years old attending the 3rd grade of elementary school who experienced difficulties with division were equally assigned to two experimental and two control groups. All participants were given a pre-test and post-test consisting of three tasks: Word Problem Task, Inverse Co-variation Task and Rules of Division Task. Children in the experimental groups were presented to problem-solving situations (exact and inexact division) in which the invariant principles underlying the concept of division were made explicit to them. These situations were accompanied by discussions and explanations on the role of the remainder, the importance of maintaining the equality of the parts and the inverse co-variation between the size of the parts and number of parts. The children in the experimental groups did significantly better on the post-test than the pre-test. The same did not occur with those in the control groups, who continued exhibiting difficulties in solving the tasks. The conclusion was that children can overcome their difficulties when the invariant principles underlying the concept of division are explicitly mentioned and associated to the problem-solving process. Educational implications are discussed.

Keywords: Children, difficulties with division, invariant principles, intervention study.

INTRODUCTION

Understanding the concept of division is often confused with skill in operating algorithms, which thus becomes the only criterion for defining and assessing a child's comprehension regarding this concept. This way of conceiving division has implication in teaching situations and appears not to assist in overcoming the main difficulties that children experience with such a complex concept. Actually, a psychological comprehension of the way children deal with mathematical problems is needed to enable creating teaching situations that promote an effective understanding of this concept. According to Vergnaud (1990, 1997), a psychological comprehension of mathematical concepts requires considering the situations that make the concept meaningful, the operational invariants that characterize a given concept, and the representation used by individuals when dealing with those situations.

In the case of division, children must deal with the operational invariants that govern this concept: (i) the size of the parts must be the same for all the parts; (ii) the size of the whole is the number of parts multiplied by the size of the parts plus the remainder; (iii) there is an inverse co-variation between the size of the parts and the number of parts; (iv) the whole must be completely distributed until the remaining elements are insufficient for further distribution; and (v) the remainder cannot be larger than or equal to the number of parts or the size of the parts (Fischbein, Deri, Nello and Marino, 1985; Harel and Cofrey, 1994; Kouba,1989; Nunes and Bryant, 1996).

Three main difficulties children experience when

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solving division problems are often documented in the literature: (i) difficulties related to the types of problems (Brown, 1981; Correa, Nunes and Bryant, 1998; Downton, 2009; Fischbein, Deri, Nello and Marino, 1985; Nesher, 1988; Skoumpoudi and Sofikiti, 2009); (ii) difficulties in understanding the inverse co-variation between the terms when the dividend remains constant (Correa, Nunes and Bryant, 1998; Kornilaki and Nunes, 1997, Squire and Bryant, 2002); and (iii) difficulties in dealing with the remainder (Carraher and Schliemann, 1991; Campbell and Fraser, 1997; Desforges and Desforges, 1980; Li and Silver, 2000; Silver, 1988; Silver, Shapiro and Deutsch, 1993; Spinillo and Lautert, 2002, 2006). The nature of these difficulties is briefly discussed below, with particular emphasis on the inverse co-variation between the terms and inappropriate ways of dealing with the remainder, as these two types of difficulties were the target of the intervention offered to the children in the present study.

Difficulties related to the type of division problem

Two types of division problems are mentioned in the literature (Fischbein, Deri, Nello and Marino, 1985; Greer, 1992; Squire and Bryant, 2002): partitive and quotitive. In partitive problems, an initial amount and the number parts in which this amount is divided are given, and it is necessary to find out the size of each part (example: “Charles bought 15 pencils to give to three of his friends. How many pencils will each friend get?”). In quotitive problems, an initial amount and the size of each part (quota) are given, and it is necessary to find out the number of parts in which the total is divided (example: “Charles bought 15 pencils. He wants to give three pencils to each friend. How many friends will get the pencils he bought?”). Studies conducted with children has shown that partitive problems are easier than quotitive problems because they involve the action schema of sharing, which is a notion children understand from an early age that has its origin in diverse social situations. On the other hand, young children seem to have less experience with quotitive problems, which appears to be acquired later through formal instruction in the school context (e.g. Anghileri, 1995; Correa, Nunes and Bryant, 1998; Fischbein, Deri, Nello and Marino, 1985; Kouba, 1989; Skoumpoudi and Sofikiti, 2009; Squire and Bryant, 2002).

Difficulties with the inverse co-variation

Children can deal with different arithmetic situations long before they are formally instructed in school. In the case of division, the idea of sharing emerges very early, as mentioned above, and there is a strong association between sharing and the initial notion young children have about division (Anghileri, 1995; Frydman and Bryant, 1988; Kornilaki and Nunes, 1997; Skoumpoudi and Sofikiti, 2009; Squire and Bryant, 2002). According to Correa, Nunes and Bryant (1998) and Nunes and Bryant (1996), division as an operation is not the same as sharing; and the initial notion of sharing, although important, does not ensure an understanding regarding the inverse co-variation between the division terms. In sharing situations, a child divides “x” amount into “y” parts until successively using up the amount to be divided, employing procedures that involve one-to-one correspondence. However, division involves one-to-many correspondence (Piaget, 1974), requiring the child to operate with three variables. For instance, when dealing with the division of flowers among a given number of vases, the child is operating with three distinct quantities: the total number of flowers, the number of vases and the number of flowers per vase. In such a situation, it is crucial to understand the relationships between the dividend (flowers) and divisor (vases) in order to determine the quotient (number of flowers in each vase). Many of the difficulties in the solving of division problems stem from not grasping the inverse co-variation between the division terms; i.e., the understanding that the greater the number of parts (vases) in which the whole is divided (total number of flowers), the smaller the size of each part (flowers per vase or quota) (e.g., Correa, Nunes and Bryant, 1998; Kornilaki and Nunes, 1997; Lautert and Spinillo, 2004, Spinillo and Lautert 2006; Nunes and Bryant, 1996; Squire and Bryant, 2002). Thus, the understanding of this inverse relation is a crucial step in the understanding of division as an operation that goes beyond the action of sharing.

Difficulties with the remainder

Children have difficulties understanding the meaning of the remainder in division problems. Some of these difficulties are due to the fact that there are many different forms of representing it (for example, the result of $50 \div 4$ may be expressed as 12.5, 12 1/2 or even 12 R2); and also due to the fact that the ways of expressing the remainder are not always incorporated into the solution of the problem (Li and Silver, 2000; Silver, 1988; Silver, Mukhopadhyay and Gabriele, 1992; Silver, Shapiro and Deutsch, 1993).

Selva (1998) examined whether six to eight-year-old children’s strategies in solving exact and inexact division word problems varied in function of the types of problems (partitive and quotitive) and the available material (objects and pencil and paper). It was found that 42% of the six-year-olds, 25% of the seven-year-olds and 6% of the eight-year-olds did not include the remainder in their solving procedures. Among the solving procedures that

Table 1. Mean and standard deviation of ages and performance on word problem task presented in the pre-test

	Group A				**Group B**			
	Control Group ($CG_{A)}$) (n = 30)		Experimental Group ($EG_{A)}$) (n = 29)		Control Group ($CG_{B)}$) (n = 20)		Experimental Group ($EG_{B)}$) (n = 21)	
	M	SD	M	SD	M	SD	M	SD
Age (months)	116	7	122	7	118	9	117	10
Word Problem	0.00	0.00	0.00	0.00	0.20	0.11	0.17	0.08

effectively mentioned the remainder there was a tendency to remove the remainder from the solution procedure; to insert the remainder in one of the parts or distribute it among the parts, resulting in parts of different sizes (violating the invariant principle related to the equality of the parts). The data showed that the material used (pencil and paper or objects) influenced the way children dealt with the remainder, especially among the six-year-olds, for whom the idea of distributing the remainder among the parts was more associated to the use of pencil and paper than concrete material.

Thus, the non-understanding of inverse co-variation and inappropriate ways of dealing with the remainder seem to be the cause of the main difficulties children have when solving division problems. One may wonder whether children might overcome these difficulties and develop an understanding of division if they had a concentrated experience with problem-solving situations in which the invariant principles of division were made explicit to them. For instance, situations of solving division problems could be accompanied by discussions and explanations on the role of the remainder, the importance of maintaining the equality of the parts and the inverse co-variation between the size of the parts and number of parts. This idea was tested in an intervention study involving elementary school children who experienced difficulties with the concept of division. The proposed intervention was based on making explicit to the child the invariant principles underlying the concept of division.

METHOD

Participants

One hundred low-income children (47 boys and 53 girls; mean age: 9 years 10 months; standard deviation: 8 months; age range: 8 years 4 months to 11 years 4 months) attending the 3rd grade of elementary school in the city of Recife, Brazil took part in this study. They were assigned to two groups based on scores obtained on a word problem task that served as a pre-test. The task consisted of the solving of ten division word problems. Group A was made up of 59 children who received a score of zero on all problems and Group B was made up of 41 children whose scores ranged from 1 to 5 (maximum of 50% of correct responses). Each group was then randomly subdivided into an Experimental Group and Control Group (Table 1).

Procedure and experimental design

The pre-test was administered individually to all participants and consisted of three tasks: word problem task, co-variation task and rules-of-division task. The word problem task was presented in the first two sessions, in which the children's performance served for the formation of the groups based on their difficulty with the concept of division (Group A and Group B). The inverse co-variation task was presented in the third session and the rules-of-division task in the fourth session. On the three tasks, the children were asked to solve partitive and quotitive problems involving exact and inexact division. The order of the presentation of the problems in each task was randomly assigned to each child.

The tasks and procedure in the post-test were the same as those on the pre-test. There was an interval of nine to ten weeks between the pre-test and post-test, with the sessions audio recorded and transcribed for subsequent analysis.

Only the children in the experimental groups (EG_A and EG_B) participated in the intervention, which consisted of two individual sessions with an interval of three to four days. The first session involved activities that focused in the inverse co-variation between the divisor and quotient when the dividend was maintained constant. The second session comprised activities that focused on the invariant principles of division as a whole, especially on the role played by the remainder in inexact division problems. The two sessions were audio recorded and transcribed.

The tasks

The *word problem task* served as a pre-test and consisted of the solving of five partitive division problems (three problems of exact division and two of inexact division) and five quotitive division problems (three problems of exact division and two of inexact division). The problems were written presented one per page in a booklet, with sufficient blank space for the child to write. Pencil and paper were available. Examples:

Partitive problem (inexact division)**:** Grandma went to the bookstore and bought 25 pencils to give to her seven grandchildren. She wants each grandchild to get the same number of pencils. How many pencils will each grandchild get?

Quotitive problem (exact division): Mary bought 35 sweets. She wants to give five sweets to each of her friends. How many friends will get the sweets she bought?

The *inverse co-variation task* consisted of the solving of three partitive and three quotitive problems – all of exact division. Each problem was read by the examiner and was written on a card placed in front of the child. Pencil and paper were available. Examples:

Partitive problem: Ann and Elizabeth went to a flower shop and each bought 21 roses. Ann wants to put her roses in three vases and Elisabeth wants to put her roses in seven vases. Who will have vases with more roses, Ann or Elizabeth?

Quotitive problem: Mark and Paul went to a toy store and each bought 18 marbles. Marcos wants to keep three marbles in each box and Paul wants to keep six marbles in each box. Who will need more boxes to keep all the marbles, Paul or Mark?

The correct answer was associated to the first character in half of the problems and the second character in the other half of the problems.

The *rules-of-division task* consisted of solving three partitive and three quotitive problems of inexact division. Each problem was written on a card placed in front of the child. One by one, each problem jointly read by the examiner and the child. At the same time, two cards were presented: one with a correct solution procedure and the other with an incorrect solution procedure. For example, the following partitive problem was presented: “A boy won 33 football player stickers and he had five envelopes. He wanted to put the same number of stickers in each envelope. How many stickers is he going to put in each envelope?” The child was then told that “Two children from another school solved this problem. Mary solved the problem in the way shown on this card (Figure 1) and John solved it in this way (Figure 2). Who solved it incorrectly: Mary or John?” The correct response was associated to the first character in half of the problems and to the second character in the other half of the problems.

Three types of mistakes were presented in the cards on this task: (i) violation of the principle of equality between the parts; (ii) remainder larger than number of parts or the size of the parts; (iii) addition of an extra part in which the remainder was included (Figures 1 e 2).

The intervention

During the intervention the problems were presented in written and jointly read by the examiner and the child. Pencil, paper and concrete material (toy cars, marbles, boxes, pencils, pencil cases, cups, trays etc.) were provided.

The child first solved the problem. Then, in a clinical interview, explanations were asked regarding the way in which he/she had solved it. In this dialogue with the examiner, the child was encouraged to present an explanation to the solution he/she had adopted and also had the opportunity to discuss how he/she had handled the remainder (when the problem involved inexact division) and the inverse relationship between the parts and the size of the parts. Then, the examiner provided feedback and comments on the strategies the child adopted, independently whether they were correct or not. Special emphasis was given to the invariant principles underlying the concept of division in these comments: discussions and explanations on the role of the remainder, the importance of maintaining the equality of the parts and the inverse co-variation between the size of the parts and number of parts. Four problem-solving activities were presented in two sessions, as described below:

First session

The focus was on inverse co-variation between division terms, aiming to assist the child in understanding that an increase in the number of parts would lead to a decrease in the size of the parts and vice-versa. Partitive and quotitive problems were presented and always involved exact division.

Activity 1

The problems were presented with two variations: one consisted of decreasing and the other consisted of increasing the divisor while maintaining the dividend constant. Example:

Partitive problem: Paul bought 24 toy cars and wants to put them in four boxes. He wants each box to have the

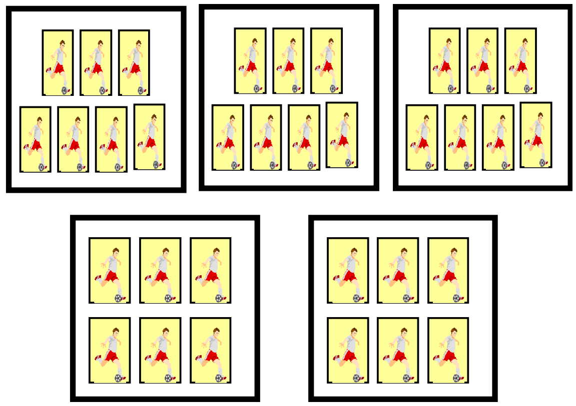

Figure 1. Incorrect solution (violation of the equality of the parts) presented in one of the cards showed to the child in the rules-of-division task

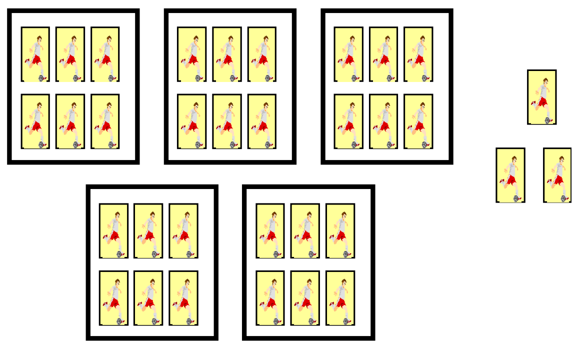

Figure 2. Correct solution presented in one of the cards showed to the child in the rules-of-division task

same number of cars. How many cars will be in each box?

Variation 1 (increase in divisor): Paul decided to increase the number of boxes. He doesn't want to put the cars in four boxes anymore. He now wants to put them in six boxes. So, he increased the number of boxes. Will the number of cars in each box go up or down? Why?

Variation 2 (decrease in divisor): Paul decided to decrease the number of boxes. He doesn't want to put the cars in six boxes anymore. He now wants to put them in two boxes. So, he decreased the number of boxes. Will the number of cars in each box go up or down? Why?

Activity 2

The problems involved two persons dividing the same quantity of objects in "x" number of parts (partitive division) or dividing the same quantity of objects in pre-established quotas (quotitive division). Examples:

Partitive problem: Richard and Joanna went to a stationery shop and each bought 30 coloured pencils. Richard wants to put his coloured pencils in 5 pencil cases and Joanna wants to put hers in 6 pencil cases. Whose pencil cases will have more coloured pencils in them, Joanna's or Richard's? Why?

Quotitive problem: Charles and Robert went to a toy store and each bought 42 toy cars. Charles wants to put 6 cars in each box and Robert wants to put 7 cars in each box. Who will need more boxes, Charles or Robert? Why?

In both activities the child was asked to explain the strategies used to solve the problems. In turn, the examiner made the inverse relationships between division terms explicit to the child, explaining that: when the total number of objects remains the same and the size of the parts (or the number of parts) increases, thus the number of parts (or the size of the parts) decreases.

Second session

The focus was on the remainder and the need to main-

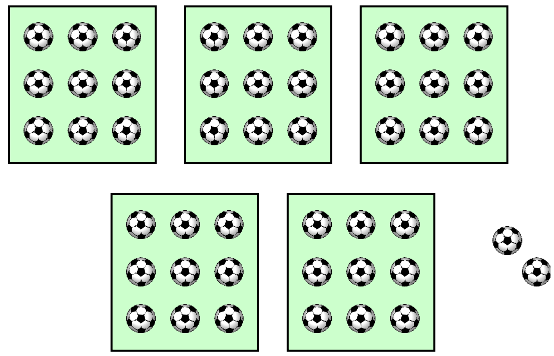

Figure 3. Correct solution presented in one of the cards showed to the child during the intervention

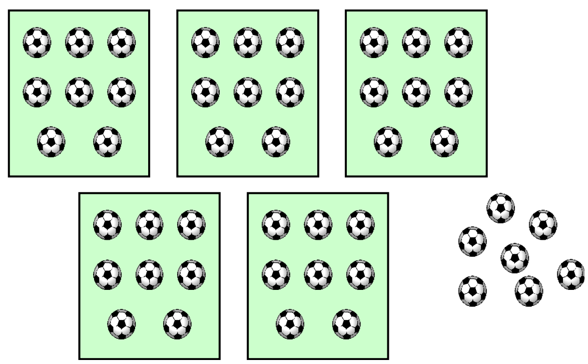

Figure 4. Incorrect solution (the remainder is larger than the number of parts) presented in one of the cards showed to the child during the intervention

tain the equality among the parts, aiming to assist the child in understanding the role of the remainder in problems of inexact division. The invariants principles of division emphasized, making explicit to the child (i) the need to maintain equality among the parts, (ii) the need to redistribute the elements of the remainder such that it never results in a greater amount or amount equal to the number of the parts or size of the parts; and (iii) the need to consider the remainder as part of the quantity that was initially divided and should therefore be included in the solution procedure. The problems presented were partitive and quotitive. The activities presented in this session were similar to the procedure employed in the previously described rules-of-division task.

Activity 3

A problem written on a card was presented, along with two cards representing two solution procedures – one with a correct procedure and one with an incorrect procedure. The instructions given by the examiner can be summarized as follows: “I gave a problem to two children to solve. One child was Ann and the other was Paul. These cards are like photographs of the way they solved the problem. I’m going to show you the problem they had to solve and you will tell me who solved it better, Ann or Paul.” Explanations were asked after each answer. Example:

Partitive problem: Jack has 47 plastic football balls and wants to put them in five boxes. He wants each box to have the same amount. How many football balls will he put in each box? (Figures 3 e 4)

After the child’s explanation, the examiner explained the reason why one procedure was more adequate than the other, making explicit the type of mistake presented in the card and the invariant principles that were violated. It is important to stress that the examiner’s explanations were always done in a context where the child was actively involved in the discussion.

Table 2. Means of correct responses in experimental and control groups on pre-test and post-test tasks (standard deviation in parentheses)

	Group A (pre-test word problem scores = 0)				Group B (pre-test word problem scores < 0.50)			
	Control CG_A (n = 30)	Experimental EG_A (n = 29)			Control CG_B (n = 20)	Experimental EG_B (n = 21)		
			df	t			df	t
Pre-test								
Inverse covariance task	0.27 (0.28)	0.39 (0.31)	57	-1.48	0.41 (0.29)	0.25 (0.27)	39	1.84
Rules-of-division task	0.52 (0.30)	0.58 (0.30)	57	-0.81	0.61 (0.26)	0.48 (0.25)	39	1.66
Post-test								
Word problem	0.08 (0.14)	0.33 (0.24)	44	-5.07**	0.18 (0.19)	0.45 (0.25)	39	-3.84**
Inverse covariance task	0.36 (0.29)	0.73 (0.34)	57	-4.50**	0.35 (0.33)	0.69 (0.32)	39	-3.33**
Rules-of-division task	0.52 (0.36)	0.99 (0.03)	29	-7.20**	0.63 (0.34)	0.98 (0.05)	20	-4.62**

Note: t-test: *p < 0.05; ** p < 0.01

Activity 4

This was a variation of the previous activity, with the difference being that only incorrect solution procedures were presented in the cards. The child was asked to identify the type of mistake presented in the incorrect procedure and indicate more appropriate ways of solving the problem. The problems were similar to those presented in Activity 3 and the incorrect procedures included the same types of mistakes. The instructions given can be summarized as follows: “I gave this division problem to a girl who attends another school: “’Carlos went to a stationery shop and bought 28 coloured pencils and wants to put them in five pencil cases. He wants each pencil case to have the same number of coloured pencils. How many coloured pencils will he put in each pencil case?’ The girl in the other school solved it wrong. She solved it in this way.” The examiner placed five pencil cases and 28 coloured pencils on the desk in front of the child and proceeded with the distribution, placing five pencils in each of two cases and six pencils in each of three cases. “I want you to find the mistake the girl made and tell me what would be the best way to solve this problem.” Explanations were asked after the child’s responses.

During the dialog between the children, he/she actively participated in the examiner’s explanations about the reason why the procedure was incorrect, the type of mistake presented and the invariant principles that were violated in the solution procedure presented in the cards.

RESULTS

Table 2 displays the scores on the pre-test and post-test. The t-test revealed no significant differences on the pre-test between the groups EG_A and CG_A or between EG_B and CG_B for the inverse covariance and rules-of-division tasks. This indicates that the performance in the experimental and control groups was the same in both Group A (100% incorrect responses) and Group B (up to 50% incorrect responses).

On the post-test, the pattern of the results was quite different from that observed on the pre-test, as significant differences were detected between the EG_A and CG_A as well as between the EG_B and CG_B on all three tasks (word problem, inverse co-variation and rules-of-division). The performance in the experimental subgroups was better than that of the control subgroups among both the children in Group A (100% incorrect responses) and those in Group B (up to 50% incorrect responses). This shows that the intervention provoked a difference

Table 3. Correlation (Pearson's r) between performance in both experimental groups on inverse co-variation task and rules-of-division task on pre-test with performance on post-test word problem task

		Word problem post-test
Experimental Group (EG_A)		
n=29		
Inverse covariance task	r	0.02
	p	0.90
Rules-of-division task	r	-0.04
	p	0.84
Experimental Group (EG_B)		
n=21		
Inverse covariance task	r	-0.18
	p	0.44
Rules-of-division task	r	0.01
	p	0.98

Table 4. Number and percentage of children on the post-test word problem task

	Children with less than 50% of correct responses		Children 50% or more of correct responses	
	N	%	N	%
Group A				
Control (**CG_A**)	29	97	1	3
Experimental **(EG_A)**	19	65	10	35
Group B				
Control (**CG_B**)	17	85	3	15
Experimental **(EG_B)**	10	48	11	52

between the groups, favouring the performance of the children in the experimental groups.

It is important to stress that the positive effect of the intervention was not the same in relation to all tasks. The intervention had a greater effect on the performance on the rules-of-division task than on the inverse co-variation task in both the EG_A ($t = -4.11$, $df = 28$, $p < 0.01$) and EG_B ($t = -4.20$, $df = 20$, $p < 0.01$) on the post-test (Table 2). Thus, the intervention seems to have been generally more effective in promoting the rules-of-division than the relations of inverse co-variation between division terms.

As the results demonstrate that it is possible to improve children's performance in the solution of division problems through the intervention carried out, one may wonder if such progress could also be related to the initial knowledge that the children already showed regarding inverse co-variation and rules-of-division. Thus, it was examined whether there was a positive correlation between the performance on the pre-test tasks (inverse covariance and rules-of-division) and the word problem task on the post-test (Table 3).

As it can be seen, children's initial knowledge on both the inverse co-variation task and rules-of-division task did not significantly correlate to the performance on the post-test word problem task. Thus, regardless of their initial knowledge of division, children in both experimental groups benefited from the intervention, even those whose score in the pre-test was zero.

Although the data are clear with regard to the gains in the experimental groups following the intervention, a further effort was made to examine to what extent the intervention assisted each child individually regarding the application of recently-acquired knowledge in the solution of division word problems. For such, the scores obtained by each child on the post-test word problem task were explored, leading to the formation of two groups – one group of children who correctly solved 50% or more of the problems and another group who correctly solved less than 50% of the problems on this task (Table 4).

The results of McNemar's test demonstrated that no significant changes occurred in Control Group A ($p = 1.0$) or Control Group B ($p = 0.25$). However, the change in the individual performance of the children was significant in both Experimental Group A (p = 0.002) and Experimental Group B (p = 0.001). Half of the children in Experimental Group B and one third of the children in Experimental Group A correctly solved 50% or more of the division problems on the post-test.

DISCUSSION AND CONCLUSIONS

The main conclusion derived from this study is that children can overcome their difficulties regarding the concept of division when the invariant principles underlying this concept were made explicit to them and associated to their mistakes when solving exact and inexact division problems. It does not mean that an intervention of this nature is the only way to promote understanding about division in children who have difficulties with this concept; but, without a doubt, the proposed intervention was able to contribute to the overcoming of the difficulties experienced by these children.

The intervention offered to the experimental groups benefited both the children who solved some division problems, albeit with difficulty, as well as those with considerable difficulty who were unable to solve any of the problems presented on the pre-test. However, children with fewer difficulties benefited more from the intervention than those with more accentuated difficulties. On the whole, the children in the two experimental groups showed progress that was not observed among those in the control groups.

Two of the main difficulties children experience with division were focused in the intervention offered to children in the experimental groups: the role played by the remainder in the solution procedure and the inverse co-variation between the division terms. Despite the improvement observed in both experimental groups, the intervention did not have the same effect on the different kinds of difficulties, as it was easier to overcome difficulties in dealing with the remainder than in understanding the inverse co-variation between division terms. One possible explanation for this is that the remainder refers to a set of elements that can be clearly represented by objects, drawings etc., while co-variation refers to relationships that are the result of mental operations and cannot be materially or graphically represented in the same way as the remainder. It therefore appears easier to take the remainder as the object of reflection than the inverse relationships between division terms.

Theorizing a little with regard to the facilitating role of the intervention in the present study, two aspects that, according to our analysis, allowed progress in the understanding of the concept of division should be addressed. One is the nature of the interaction between the examiner and child, and the other is the nature of the concept of division. With regard to the first aspect, it is worth noting that the examiner provided discussions that led the child to verbalize and reflect on his/her own problem-solving processes when faced with a given situation-problem. Moreover, the interaction was marked by both the child and examiner demanding the explicit detailing of the justifications that sustained the forms of reasoning. Specifically regarding the concept of division, it is important to stress that the intervention was based on two important instances of the theoretical perspective proposed by Vergnaud (1990, 1997) concerning mathematical concepts: situations and invariants. The intervention put into action the idea that a single situation is not enough to encompass all the characteristics of a concept; it is necessary to examine the same concept in the light of diverse situations – represented here by the different activities proposed in the intervention involving the invariant principles of division. In turn, these principles were related to the types of difficulties children experience with the concept, as documented in the literature. The proposed intervention transformed the children's errors into situations of learning.

This study clearly has some educational implications for the teaching of division in elementary school. The tasks proposed in the intervention could be adapted to the classroom setting. Moreover, the teaching of division can be introduced with problems of both exact and inexact division. Actually, the remainder showed to be an important recourse to help children to deal with the invariant principles that govern the concept of division. Also, it seems important to promote classroom discussion: talk to children about their errors and the strategies they use (independently of whether they are correct or not); by encouraging them to communicate verbally their ways of thinking. Discussing children's difficulties and emphasizing these principles seem to generate an appropriate understanding of division.

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