Full Length Research Paper

An Investigation of the Collatz Conjecture Sequence

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Abstract

An investigation into the Collatz conjecture sequence a_0, a_1, a_2, ... a_x, where a0 is an element of the set P, and P is a family of subsets that belong to the partition of the set $\{1,3,5,7,...\}$. The investigation finds that there exist a family of subsets T that belong to P where the Collatz Conjecture sequence a_0, a_1, a_2, ... a_x, satisfies the inequality $a_0 > a_x$.

Keywords: monotonic lattice, catalan numbers, directed graph, and combinatorial convergent summation.

INTRODUCTION

1937 Mr. L. Collatz posed a conjecture also known as the mapping problem, Hasse's algorithm, or Katutani's problem which states. Let a0 be a positive integer if iterating:

$$a_x = \begin{cases} g(a_{x-1}) = \dfrac{1}{2} a_{x-1} & \text{for } a_{x-1} \in \{2n : n \in N\} \\ f(a_{x-1}) = 3a_{x-1} + 1 & \text{for } a_{x-1} \in \{2n-1 : n \in N\} \end{cases}$$

Then a_0 will always iterate a_x: $a_x = 1$.

Example 1: let $a_0 = 159$

$$159 \xrightarrow{f} 478 \xrightarrow{g} 239 \xrightarrow{f} 718 \xrightarrow{g} 359 \xrightarrow{f} 1078 \xrightarrow{g} 539 \xrightarrow{f} 1618$$

$$\xrightarrow{g} ... \xrightarrow{g} 5 \xrightarrow{f} 16 \xrightarrow{g} 8 \xrightarrow{g} 4 \xrightarrow{g} 2 \xrightarrow{g} 1$$

The members of the Collatz conjecture sequence are also known as the hailstone numbers.

Mathematical Topics:

Number /set theory, graph theory, and combinatorics,

Symbols and notations

N - Set of natural number starting with 1

N_0 - Set of whole numbers starting with 0

$\xrightarrow{f}$ - The function f(x) =3x+1

$\xrightarrow{g}$ - The function g(x) = 0.5x

$\xrightarrow{fg}$ - The function g(f(x))

$\subset$ - Subset that belongs to

$\coprod$ - Disjoint set union

$\cup$ - Set Union

$\cap$ - Set intersection

iff - If and only if

$\Leftrightarrow$ - If and only if

$\forall$ - For all

$\exists$ - There exist

$\wedge$ - Boolean And

$\vee$ - Boolean Or

$\neg$ Boolean not

$\Rightarrow$ - It implies

M - Directional graph

$M(k)$ - Directional subset graph of M

Cm - Catalan Number