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Full Length Research Paper

An alternate state space approach to study the effect of heat source in generalized Thermo-elasticity

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In infinite rotating media the fundamental equations of the problems of generalized thermo-elasticity with one relaxation time including heat sources have been written in the form of a vector-matrix differential equation in the domain of Laplace Transformation for one-dimensional problem. These equations have been solved by a different approach. Here we take two types of boundary conditions; namely a) a step input of temperature and zero stress and b) a step input of stress and zero temperature. The results have been compared to those available in the open literature. The corresponding graphs have been drawn.

Keywords: State Space Approach, Heat Source, Generalized Thermoelasticity Subject Classification.

INTRODUCTION

In the linear dynamical theory of classical thermo-elasticity the governing equations for displacement and temperature fields consist of the coupled partial differential equation of motion and the Fourier's law of heat conduction equation. For displacement field, the equation is governed by a wave type hyperbolic equation, whereas that for the temperature field, it is governed by a diffusion type parabolic equation. From this we can remark that the classical thermo-elasticity predicts a finite speed for predominantly elastic disturbances but an infinite speed for predominantly thermal disturbances, which are coupled together. This means that a part of every solution of the equations extends to infinity (Lord and Shulman, 1967). Experimental investigations by (Ackerman et al., 1966, Ackerman and Guyer, 1968 and Ackerman and Overton), von Gutfeld and Nethercot (1996); Taylor et al., (1969), Jackson and Walker (1971), and many others, conducted on different solids, have shown that heat pulses do not propagate with infinite speeds. In order to overcome this paradox, efforts were made to modify classical thermo-elasticity, on different grounds, for obtaining a wave type heat conduction equation (Kaliski, 1965; Norwood and Warren, 1969; Norwood and Warren, 1969 and Green and Lindsay, 1972). A comprehensive list on this generalization for the last two decades is available in the works of (Green and Lindsay, 1972 and Chandrasekharaiah, 1998).

At present there are three generalizations of the classical theory of elasticity: the first proposed by (Lord and Shulman, 1967) (L-S theory) involves one relaxation time for a thermo-elastic process, the second by (Norwood and Warren) (G-L theory), which takes into account two parameters on relaxation times and the third proposed by (Green and Naghdi 1995, Green and Naghd, 1995 and Green and Naghdi, 1995). Owing to the mathematical complexities encountered in coupled thermo-elasticity, mainly due to inertia and coupling terms in governing equations, these types of problems are mostly confined to one-dimensional problems (Suhubi, 1975; Lebon, 1982; Chandrasekharaiah and Narasimha, 1993, Furukawa et al., 1990 and Anwar and Sherief, 1988)

However, in the present paper following one parameter L-S theory, the authors have considered a problem of heat sources distributed over a plane area in infinite isotropic elastic solid. The alternate state space approach of Hetnarski (1964) has been considered for the analysis of the present problem.

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Basic equations and formulations of the problem

An isotropic, homogeneous, thermally conducting elastic medium with density λ and Lame constants λ and μ bounded by the planes x = 0 and x = L with a heat source distributed over the plane area is considered.
The equations of motion in the absence of body forces are

$$\tau_{ij,j} = \rho \ddot{u}_i \quad (2.1)$$

where

$$\tau_{ij} = \lambda\Delta\delta_{ij} + 2\mu u_{i,j} - \beta(T + t_1\dot{T})\delta_{ij} \quad (2.2)$$

$$\Delta = u_{i,j} \quad (2.3)$$

$$\beta = (3\lambda + 2\mu)\alpha \quad (2.4)$$

The heat conduction equation is

$$q_{i,i} = -\rho c_e(\dot{T} + t_0\ddot{T}) - \beta T_0\dot{\Delta} \quad (2.5)$$

Where

$$q_i = -KT_{,i} \quad (2.6)$$

all the terms have the same significance as in (Lord and Shulman 1967)
Combining (2.5) and (2.6) and adding the source term, we obtain

$$KT_{,ij} = \rho c_e(\dot{T} + t_0\ddot{T}) + \beta T_0\dot{\Delta} - (1 + t_0\frac{\partial}{\partial t})Q \quad (2.7)$$

Combining (2.1), (2.5) and (2.6) we obtain the displacement equation of motion as

$$\rho\ddot{\vec{u}} = (\lambda+\mu)\nabla(\nabla.\vec{u}) + \mu\nabla^2\vec{u} - \beta\nabla(T + t_1\dot{T})\delta_{ij} \quad (2.8)$$

where
k is the thermal conductivity,
c_e is the specific heat,
α is the coefficient of thermal expansion,
t_0 and t_1 are the thermal relaxation times $t_1 \geq t_0 \geq 0$,
$\delta_{ij} = 1$, $t_1 = 0$ for the L-S theory and $\delta_{ij} = 0$, $t_1 > 0$ for the G-L theory.
Since we are dealing with an isotropic medium, without any loss of generality, we may consider the waves propagating in the x-direction. All field variables are supposed to be functions of x and t only.
From (2.7) and (2.8) we get

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} - \rho\frac{\partial^2 u}{\partial t^2} = \beta(\frac{\partial T}{\partial x} + t_1\frac{\partial^2 T}{\partial x\partial t}) \quad (2.9)$$

$$K\frac{\partial^2 T}{\partial x^2} = \rho c_e(\frac{\partial T}{\partial t} + t_0\frac{\partial^2 T}{\partial t^2}) + \beta T_0(\frac{\partial \dot{u}}{\partial x} + \delta_{1k}t_0\frac{\partial \ddot{u}}{\partial x}) - (1 + t_0\frac{\partial}{\partial t})Q \quad (2.10)$$

We take the initial conditions as

$$u = 0, T = 0, \frac{\partial u}{\partial t} = 0, \quad \frac{\partial T}{\partial t} = 0 \text{ at } t = 0, x \geq 0 \quad (2.11)$$

We now define the quantities as in (Sharma 1997)

$$\eta = \frac{x}{L}, \quad \tau = \frac{kt}{L^2}, \quad Z = \frac{T}{T_0}, \quad U = \frac{\rho v^2 u}{\beta T_0 L}, \quad \tau_0 = \frac{kt_0}{L^2}$$

$$\tau_1 = \frac{kt_1}{L^2} \quad k = \frac{K}{\rho c_e}, v^2 = \frac{\lambda + 2\mu}{\rho}, \quad \varepsilon = \frac{\beta^2 T_0}{\rho^2 c_e v^2} \quad (2.12)$$

By using equation (2.12), equations (2.9) and (2.10) can be written as

$$\frac{\partial^2 U}{\partial\eta^2} - b\frac{\partial^2 U}{\partial\tau^2} = \frac{\partial Z}{\partial\eta} + \tau_1\frac{\partial^2 Z}{\partial\eta\partial\tau}$$

(2.13)

$$\frac{\partial^2 Z}{\partial\eta^2} - (\frac{\partial Z}{\partial\tau} + \tau_0\frac{\partial^2 Z}{\partial\tau^2}) = \varepsilon(\frac{\partial^2 U}{\partial\eta\partial\tau} + \delta_{1k}\tau_0\frac{\partial^3 U}{\partial\eta\partial\tau^2}) - (1 + \tau_0\frac{\partial}{\partial\tau})Q \quad (2.14)$$

where $b = \frac{k^2}{v^2 L^2}$

The initial conditions (2.11) become

$$U(\eta,0) = 0, Z(\eta,0) = 0, \frac{\partial U}{\partial\tau} = 0, \frac{\partial Z}{\partial\tau} = 0 \quad (2.15)$$

and the inequality for relaxation times become $\tau \geq \tau_0$ (2.16)

Now, if we consider the problem of a semi-infinite medium, then $L \to \infty$.

$\therefore b \to 0$ and equations (2.13) and (2.14) take the following form

$$\frac{\partial^2 U}{\partial\eta^2} - b\frac{\partial^2 U}{\partial\tau^2} = \frac{\partial Z}{\partial\eta} + \tau_1\frac{\partial^2 Z}{\partial\eta\partial\tau}$$

(2.17)

$$\frac{\partial^2 Z}{\partial\eta^2} - (\frac{\partial Z}{\partial\tau} + \tau_0\frac{\partial^2 Z}{\partial\tau^2}) = \varepsilon(\frac{\partial^2 U}{\partial\eta\partial\tau} + \delta_{1k}\tau_0\frac{\partial^3 U}{\partial\eta\partial\tau^2}) - (1 + \tau_0\frac{\partial}{\partial\tau})Q \quad (2.18)$$

If $Q \to 0$, the equations (2.17) and (2.18) are exactly the same as deduced in [28].We assume that the heat source acts on the plane $x = 0$ and is of the form $Q = Q_0\delta(\eta)H(\tau)$, where Q_0 is a constant, $\delta(\eta)$ and $H(\tau)$ are the Dirac delta function of η and the Heaviside unit step function of τ respectively.

We now apply the Laplace transform of parameter p defined as

$$\overline{U}(\eta, p) = \int_0^\infty U(\eta,\tau)e^{-p\tau}d\tau$$

$$\overline{Z}(\eta, p) = \int_0^\infty Z(\eta,\tau)e^{-p\tau}d\tau$$

(2.19)

Taking the Laplace transform on both sides of equations (2.17), (2.18), we obtain

$$\frac{d^2\overline{U}}{d\eta^2} = (1+\tau_1 p)\frac{d\overline{Z}}{d\eta}$$

(2.20)

$$\frac{d^2\overline{Z}}{d\eta^2} = p(1+\tau_0 p)\overline{Z} + \varepsilon p(1+\delta_{1k}\tau_0 p)\frac{\partial\overline{U}}{\partial\eta} - (1+\tau_0 p)\frac{Q_0\delta(\eta)}{p} \qquad (2.21)$$

For L-S theory, equations (2.20) and (2.21) take the following form:

$$\frac{d^2\overline{U}}{d\eta^2} = \frac{d\overline{Z}}{d\eta}$$

(2.22)

$$\frac{d^2\overline{Z}}{d\eta^2} = p(1+\tau_0 p)[\overline{Z} + \varepsilon\frac{\partial\overline{U}}{\partial\eta} - \frac{Q_0\delta(\eta)}{p^2}] \qquad (2.23)$$

For G-L theory, equations (2.20) and (2.21) take the following form

$$\frac{d^2\overline{U}}{d\eta^2} = (1+\tau_1 p)\frac{d\overline{Z}}{d\eta} \qquad (2.24)$$

$$\frac{d^2\overline{Z}}{d\eta^2} = p(1+\tau_0 p)[\overline{Z} + \varepsilon g\frac{\partial\overline{U}}{\partial\eta} - \frac{Q_0\delta(\eta)}{p^2}] \qquad (2.25)$$

where

$g = (1+\tau_0 p)^{-1}$

Now we define

$$\frac{d\overline{Z}}{d\eta} = \overline{Z}' \text{ and } \frac{d\overline{U}}{d\eta} = \overline{U}' \qquad (2.26)$$

So, the equations (2.20) and (2.21) reduce to

$$\frac{d\overline{U}'}{d\eta} = \tau_1^*\overline{Z} \qquad (2.27)$$

$$\frac{d\overline{Z}'}{d\eta} = p\tau_0^*\overline{Z} + p\varepsilon\tau_0'\overline{U}' - \frac{\tau_0^* Q_0\delta(\eta)}{p}$$

(2.28)

where

$\tau_0^* = (1+\tau_0 p)$, $\tau_0' = (1+\delta_{1k}\tau_0 p)$, $\tau_1^* = (1+\tau_1 p)$

(2.29)

In particular for L-S theory $\tau_0' = (1+\tau_0 p)$ as $\delta_{1k} = 1$ and for G-L theory $\tau_0' = 1$ as $\delta_{1k} = 0$.

Using equations (2.26), (2.27) and (2.28) we get

$$\frac{d}{d\eta}\begin{pmatrix}\overline{Z}\\ \overline{Z}'\\ \overline{U}'\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\ p\tau_0^* & 0 & p\varepsilon\tau_0'\\ 0 & \tau_1^* & 0\end{pmatrix}\begin{pmatrix}\overline{Z}\\ \overline{Z}'\\ \overline{U}'\end{pmatrix} + \begin{pmatrix}0\\ -\frac{\tau_0^* Q_0\delta(\eta)}{p}\\ 0\end{pmatrix} \qquad (2.30)$$

This can be written as

$$\frac{d\vec{C}}{d\eta} = A\vec{C} + B \qquad (2.31)$$

where

$$A = \begin{pmatrix}0 & 1 & 0\\ p\tau_0^* & 0 & p\varepsilon\tau_0'\\ 0 & \tau_1^* & 0\end{pmatrix}, B = \begin{pmatrix}0\\ -\frac{\tau_0^* Q_0\delta(\eta)}{p}\\ 0\end{pmatrix} \text{ and } \vec{C} = \begin{pmatrix}\overline{Z}\\ \overline{Z}'\\ \overline{U}'\end{pmatrix} \qquad (2.32)$$

Equation (2.31) can be integrated by means of the matrix exponential to yield

$$\vec{C}(\eta, p) = e^{A\eta}[\vec{C}(0, p) + B_1] \qquad (2.33)$$

where

$$B_1 = \int e^{-A\eta}Bd\eta$$

The characteristic equation is

$$\begin{vmatrix}-m & 1 & 0\\ p\tau_0^* & -m & p\varepsilon\tau_0'\\ 0 & \tau_1^* & -m\end{vmatrix} = 0 \qquad (2.34)$$

Solving equation (2.34), we obtain

$m_1 = 0$, m_2 *and* $m_3 = \pm\sqrt{ap} = \pm m$,say (2.35)

where

$$a=\tau_0^*+\varepsilon\tau_1^*\tau_0^{/} \quad (2.36)$$

Now by Cayley-Hamilton theorem, we get

$A^3-paA=0$ (2.37)

This equation implies that A can be expressed in terms of A^2, A and I, the unit vector of order three.

$$e^{A\eta}=S_0I+S_1A+S_2A^2=D(\eta,p) \text{ (say)} \quad (2.38)$$

The scalar coefficients S_0, S_1 and S_2 of equation (2.38), are $S_0=1$,

$$S_1=\frac{1}{2m}[\exp(m\eta)-\exp(-m\eta)],$$

$$S_2=\frac{1}{2m^2}[\exp(m\eta)+\exp(-m\eta)-2] \quad (2.39)$$

Therefore, equation (2.33) reduces to

$$\vec{C}(\eta,p)=D(\eta,p)[\vec{C}(0,p)+B_1]$$

(2.40)

Where

$$D(\eta,p)=\begin{pmatrix}1+p\tau_0^*S_2 & S_1 & p\varepsilon\tau_0^{/}S_2\\ p\tau_0^*S_1 & 1+paS_2 & p\varepsilon\tau_0^{/}S_1\\ p\tau_0^*\tau_1^*S_2 & \tau_1^*S_1 & 1+p\varepsilon\tau_0^{/}\tau_1^*S_2\end{pmatrix} \quad (2.41)$$

and

$$B_1=-\frac{1}{p}\int\exp(-A\eta)\tau_0^*Q_0\delta(\eta)=-\frac{\tau_0^*Q_0}{p}$$

Therefore, equation (2.40) reduces to

$$\begin{pmatrix}\overline{Z}(\eta,p)\\ \overline{Z}'(\eta,p)\\ \overline{U}'(\eta,p)\end{pmatrix}=D(\eta,p)\left\{\begin{pmatrix}\overline{Z}(0,p)\\ \overline{Z}'(0,p)\\ \overline{U}'(0,p)\end{pmatrix}+\begin{pmatrix}0\\ -\dfrac{\tau_0^*Q_0}{p}\\ 0\end{pmatrix}\right\}$$

(2.42)

Now we take two types of boundary conditions such as

Thermal shock: $T(0,t)=T_0H(t)$, $\sigma(0,t)=0$

Normal load:

$T(0,t)=0$, $\sigma(0,t)=\sigma_0H(t)$ (2.43)

By (2.12), (2.43) reduces to

$Z(0,\tau)=T_0H(\tau)$, $\sigma^*(0,\tau)=0$ and

$Z(0,\tau)=0$, $\sigma^*(0,\tau)=\sigma_0^*H(\tau)$ (2.44)

By Laplace transform, equations (2.44) become

for thermal shock : $\overline{Z}(0,p)=\dfrac{T_0}{p}$, $\overline{U}'(0,p)=\dfrac{\tau_1^*T_0}{p}$

for normal load : $\overline{Z}(0,p)=0$, $\overline{U}'(0,p)=\dfrac{\sigma_0^*}{p}$ (2.45)

Now from (2.42), we get

$$\overline{Z}(\eta,p)=(1+p\tau_0^*S_2)\overline{Z}(0,p)+S_1[\overline{Z}'(0,p)-\frac{\tau_0^*Q_0}{p}]+p\varepsilon\tau_0^{/}S_2\overline{U}'(0,p) \quad (2.46)$$

$$\overline{Z}'(\eta,p)=p\tau_0^*S_1\overline{Z}(0,p)+(1+paS_2)[\overline{Z}'(0,p)-\frac{\tau_0^*Q_0}{p}]+p\varepsilon\tau_0^{/}S_1\overline{U}'(0,p) \quad (2.47)$$

$$\overline{U}'(\eta,p)=p\tau_0^*\tau_1^*S_2\overline{Z}(0,p)+\tau_1^*S_1[\overline{Z}'(0,p)-\frac{\tau_0^*Q_0}{p}]+(1+p\varepsilon\tau_0^{/}\tau_1^*S_2)\overline{U}'(0,p) \quad (2.48)$$

By (2.45) and (2.47), we get

for thermal shock :

$$\overline{Z}'(0,p)=-T_0\sqrt{\frac{a}{p}}-\frac{\tau_0^*Q_0}{p} \quad (2.49)$$

for normal load :

$$\overline{Z}'(0,p)=-\frac{\varepsilon\tau_0^{/}\sigma_0^*}{\sqrt{ap}}-\frac{\tau_0^*Q_0}{p} \quad \text{vff}$$

Therefore, for thermal shock

$$\overline{Z}(\eta,p)=\frac{T_0}{p}\exp(-m\eta)+\frac{\tau_0^*Q_0}{\sqrt{ap^3}}\exp(-m\eta) \quad (2.51)$$

$$\overline{Z}'(\eta,p)=-T_0\sqrt{\frac{a}{p}}\exp(-m\eta)-\frac{\tau_0^*Q_0}{p}\exp(-m\eta) \quad (2.52)$$

$$\overline{U}(\eta,p)=-\frac{\tau_1^*T_0}{\sqrt{ap^3}}\exp(-m\eta)-\frac{\tau_1^*\tau_0^*Q_0}{ap^2}\exp(-m\eta) \quad (2.53)$$

$$\overline{U}'(\eta,p)=\frac{\tau_1^*T_0}{p}\exp(-m\eta)+\frac{\tau_1^*\tau_0^*Q_0}{\sqrt{ap^3}}\exp(-m\eta) \quad (2.54)$$

and for normal load

$$\overline{Z}(\eta,p)=-\frac{\varepsilon\tau_0^{/}\sigma_0^*}{ap}[1-\exp(-m\eta)]+\frac{\tau_0^*Q_0}{\sqrt{ap^3}}\exp(-m\eta) \quad (2.55)$$

$$\overline{Z}'(\eta,p)=-\frac{\varepsilon\tau_0^{/}\sigma_0^*}{\sqrt{ap}}\exp(-m\eta)-\frac{\tau_0^*Q_0}{p}\exp(-m\eta) \quad (2.56)$$

$$\overline{U}(\eta,p)=\frac{\sigma_0^*}{ap\sqrt{ap}}[\tau_0^*\sqrt{ap}\eta-\varepsilon\tau_0^{/}\tau_1^*\exp(-m\eta)]-\frac{\tau_1^*\tau_0^*Q_0}{ap^2}\exp(-m\eta) \quad (2.57)$$

$$\overline{U}'(\eta,p)=\frac{\varepsilon\tau_0^{/}\tau_1^*\sigma_0^*}{ap}\exp(-m\eta)+\frac{\tau_0^*\sigma_0^*}{ap}+\frac{\tau_1^*\tau_0^*Q_0}{\sqrt{ap^3}}\exp(-m\eta) \quad (2.58)$$

Now for conventional coupled theory of thermo-elasticity, $\tau_1=\tau_0=0$.

$\therefore \tau_0^*=1$, $\tau_1^*=1$, $\tau_0^{/}=1$. $\therefore a=1+\varepsilon$, $m=\sqrt{(1+\varepsilon)p}$

Therefore, for thermal shock

$$\overline{Z}(\eta,p)=\frac{T_0}{p}\exp(-\sqrt{(1+\varepsilon)p}\eta)+\frac{Q_0}{\sqrt{(1+\varepsilon)p^3}}\exp(-\sqrt{(1+\varepsilon)p}\eta) \quad (2.51a)$$

$$\bar{Z}'(\eta,p)=-T_0\sqrt{\frac{(1+\varepsilon)}{p}}\exp(-\sqrt{(1+\varepsilon)p}\eta)-\frac{Q_0}{p}\exp(-\sqrt{(1+\varepsilon)p}\eta) \quad (2.52a)$$

$$\bar{U}(\eta,p)=-\frac{T_0}{\sqrt{(1+\varepsilon)p^3}}\exp(-\sqrt{(1+\varepsilon)p}\eta)-\frac{Q_0}{(1+\varepsilon)p^2}\exp(-\sqrt{(1+\varepsilon)p}\eta) \quad (2.53a)$$

$$\bar{U}'(\eta,p)=\frac{T_0}{p}\exp(-\sqrt{(1+\varepsilon)p}\eta)+\frac{Q_0}{\sqrt{(1+\varepsilon)p^3}}\exp(-\sqrt{(1+\varepsilon)p}\eta) \quad (2.54a)$$

and for normal load

$$\bar{Z}(\eta,p)=-\frac{\varepsilon\sigma_0^*}{(1+\varepsilon)p}[1-\exp(-\sqrt{(1+\varepsilon)p}\eta)]+\frac{Q_0}{\sqrt{(1+\varepsilon)p^3}}\exp(-\sqrt{(1+\varepsilon)p}\eta) \quad (2.55a)$$

$$\bar{Z}'(\eta,p)=-\frac{\varepsilon\sigma_0^*}{\sqrt{(1+\varepsilon)p}}\exp(-\sqrt{(1+\varepsilon)p}\eta)-\frac{Q_0}{p}\exp(-\sqrt{(1+\varepsilon)p}\eta) \quad (2.56a)$$

$$\bar{U}(\eta,p)=\frac{\sigma_0^*}{(1+\varepsilon)p\sqrt{(1+\varepsilon)p}}[\sqrt{(1+\varepsilon)p}\eta-\varepsilon\exp(-\sqrt{(1+\varepsilon)p}\eta)]-\frac{Q_0}{(1+\varepsilon)p^2}\exp(-\sqrt{(1+\varepsilon)p}\eta) \quad (2.57a)$$

$$\bar{U}'(\eta,p)=\frac{\varepsilon\sigma_0^*}{(1+\varepsilon)p}\exp(-\sqrt{(1+\varepsilon)p}\eta)+\frac{\sigma_0^*}{(1+\varepsilon)p}+\frac{Q_0}{\sqrt{(1+\varepsilon)p^3}}\exp(-\sqrt{(1+\varepsilon)p}\eta) \quad (2.58a)$$

Inverting the Laplace transform of equations (2.51a) to (2.58a), we get for thermal shock

$$Z(\eta,\tau)=T_0 erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)+\frac{Q_0}{\sqrt{(1+\varepsilon)}}\left[\frac{2\sqrt{\tau}}{\pi}\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)-\sqrt{(1+\varepsilon)}\eta erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)\right] \quad (2.59)$$

$$Z(\eta,\tau)=-T_0\sqrt{\frac{(1+\varepsilon)}{\pi\tau}}\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)-Q_0 erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right) \quad (2.60)$$

$$U(\eta,\tau)=-\frac{T_0}{\sqrt{(1+\varepsilon)}}\left[\frac{2\sqrt{\tau}}{\pi}\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)\right]-\sqrt{(1+\varepsilon)}\eta erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)$$

$$-\frac{2Q_0}{(1+\varepsilon)}\left[\frac{1}{\sqrt{\pi}}\left(1-\frac{\eta\sqrt{(1+\varepsilon)\tau}}{2\sqrt{\pi}}\right)\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)+\frac{\sqrt{(1+\varepsilon)}\eta}{2}erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)\right] \quad (2.61)$$

$$U'(\eta,\tau)=T_0 erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)+\frac{Q_0}{\sqrt{(1+\varepsilon)}}\left[\frac{2\sqrt{\tau}}{\pi}\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)-\sqrt{(1+\varepsilon)}\eta erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)\right] \quad (2.62)$$

and for normal load

$$Z(\eta,\tau)=-\frac{\varepsilon\sigma_0^*}{(1+\varepsilon)}erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}-1\right)$$

$$+\frac{Q_0}{\sqrt{(1+\varepsilon)}}\left[\frac{2\sqrt{\tau}}{\pi}\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)-\sqrt{(1+\varepsilon)}\eta erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)\right] \quad (2.63)$$

$$Z'(\eta,\tau)=-\frac{\varepsilon\sigma_0^*}{\sqrt{(1+\varepsilon)}}\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)-Q_0 erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right) \quad (2.64)$$

$$U(\eta,\tau)=\frac{\sigma_0^*}{\sqrt{(1+\varepsilon)^3}}\left[\sqrt{(1+\varepsilon)}\eta-\frac{2\sqrt{\tau}\varepsilon}{\pi}\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)\right]-\sqrt{(1+\varepsilon)}\eta erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)$$

$$-\frac{2Q_0}{(1+\varepsilon)}\left[\frac{1}{\sqrt{\pi}}\left(1-\frac{\eta\sqrt{(1+\varepsilon)\tau}}{2\sqrt{\pi}}\right)\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)+\frac{\sqrt{(1+\varepsilon)}\eta}{2}erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)\right] \quad (2.65)$$

$$U'(\eta,\tau)=\frac{\sigma_0^*}{(1+\varepsilon)}\left[1-\varepsilon erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)\right]$$

$$+\frac{Q_0}{\sqrt{(1+\varepsilon)}}\left[\frac{2\sqrt{\tau}}{\pi}\exp\left(-\frac{(1+\varepsilon)\eta^2}{4\tau}\right)-\sqrt{(1+\varepsilon)}\eta erfc\left(\frac{\sqrt{(1+\varepsilon)}\eta}{2\sqrt{\tau}}\right)\right] \quad (2.66)$$

In the L-S theory, as $p\to\infty$, we obtain from equations (2.51) to (2.58) for thermal shock

$$\bar{Z}(\eta,p)=\frac{T_0}{p}\exp\left(\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta)+\frac{\tau_0 Q_0}{\lambda_0 p}\exp\left(\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta) \quad (2.51b)$$

$$\bar{Z}'(\eta,p)=-T_0\lambda_0\left[1+\frac{1}{2\tau_0 p}\right]\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta)-\tau_0 Q_0\left[1+\frac{1}{\tau_0 p}\right]\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta) \quad (2.52b)$$

$$\bar{U}(\eta,p)=-\frac{T_0}{\lambda_0}\left[\frac{1}{p^2}-\frac{1}{2\tau_0 p^3}\right]\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta)-\frac{\tau_0 Q_0}{\lambda_0^2 p^2}\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta) \quad (2.53b)$$

$$\bar{U}'(\eta,p)=\frac{T_0}{p}\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta)+\frac{\tau_0 Q_0}{\lambda_0 p}\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta) \quad (2.54b)$$

and for normal load

$$\bar{Z}(\eta,p)=-\frac{\varepsilon\sigma_0^*\tau_0}{\lambda_0^2 p}\left[1-\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta)\right]-\frac{\tau_0 Q_0}{\lambda_0 p}\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta) \quad (2.55b)$$

$$\bar{Z}'(\eta,p)=-\frac{\varepsilon\sigma_0^*\tau_0}{\lambda_0}\left(1+\frac{1}{2\tau_0 p}\right)\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta)-\tau_0 Q_0\left(1+\frac{1}{\tau_0 p}\right)\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta) \quad (2.56b)$$

$$\bar{U}(\eta,p)=\frac{\sigma_0^*\tau_0\lambda_0\eta}{\lambda_0^3 p}+\frac{\varepsilon\sigma_0^*\tau_0}{\lambda_0^3 p^2}\left(1-\frac{1}{2\tau_0 p}\right)\exp\left(-\frac{\lambda\eta}{2\tau_0}\right)\exp(-\lambda_0 p\eta)$$

$$-\frac{\tau_0 Q_0}{\lambda_0^2 p^2}\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)\exp\left(-\lambda_0 p\eta\right) \quad (2.57b)$$

$$\bar{U}'(\eta,p)=\frac{\varepsilon\sigma_0^*}{(1+\varepsilon)p}\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)\exp\left(-\lambda_0 p\eta\right)+\frac{\sigma_0^*}{(1+\varepsilon)p}+\frac{\tau_0 Q_0}{\lambda_0 p}\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)\exp\left(-\lambda_0 p\eta\right) \quad (2.58b)$$

Taking Inverse Laplace transform of equations (2.51b) to (2.58b), we get for thermal shock

$$Z(\eta,\tau)=T_0 H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)+\frac{\tau_0 Q_0}{\lambda_0}H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right) \quad (2.67)$$

$$Z'(\eta,\tau)=-T_0\lambda_0\left[\delta(\tau-\lambda_0\eta)+\frac{1}{2\tau_0}H(\tau-\lambda_0\eta)\right]\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)$$

$$-Q_0\left[\tau_0\delta(\tau-\lambda_0\eta)+H(\tau-\lambda_0\eta)\right]\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right) \quad (2.68)$$

$$U(\eta,\tau)=-\frac{T_0}{\lambda_0}\left[(\tau-\lambda_0\eta)-\frac{(\tau-\lambda_0\eta)^2}{4\tau_0}\right]H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)$$

$$-\frac{\tau_0 Q_0}{\lambda_0^2}(\tau-\lambda_0\eta)H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right) \quad (2.69)$$

$$U'(\eta,\tau)=T_0 H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)+\frac{\tau_0 Q_0}{\lambda_0}H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right) \quad (2.70)$$

and for normal load

$$Z(\eta,\tau)=-\frac{\varepsilon\sigma_0^*}{(1+\varepsilon)}\left[1-H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)\right]-\frac{\tau_0 Q_0}{\lambda_0}H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right) \quad (2.71)$$

$$Z'(\eta,\tau)=-\frac{\varepsilon\sigma_0^*\tau_0}{\lambda_0}\left[\delta(\tau-\lambda_0\eta)+\frac{1}{2\tau_0}H(\tau-\lambda_0\eta)\right]\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)$$

$$-Q_0\left[\tau_0\delta(\tau-\lambda_0\eta)+H(\tau-\lambda_0\eta)\right]\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right) \quad (2.72)$$

$$U(\eta,\tau)=\frac{\sigma_0^*}{(1+\varepsilon)}\left[\eta-\frac{\varepsilon}{\tau_0}\left\{(\tau-\lambda_0\eta)-\frac{(\tau-\lambda_0\eta)^2}{4\tau_0}\right\}H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)\right]$$

$$-\frac{Q_0}{(1+\varepsilon)}(\tau-\lambda_0\eta)H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right) \quad (2.73)$$

$$U'(\eta,\tau)=\frac{\sigma_0^*}{(1+\varepsilon)}\left[1+\varepsilon H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right)\right]+\frac{\tau_0 Q_0}{\lambda_0}H(\tau-\lambda_0\eta)\exp\left(-\frac{\lambda_0\eta}{2\tau_0}\right) \quad (2.74)$$

where

$$\lambda_0=\sqrt{\tau_0(1+\varepsilon)}$$

In the case of G-L theory, we obtain from (2.51) to (2.58) for thermal shock

$$\bar{Z}(\eta,p)=\frac{T_0}{p}\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right)+\frac{a_0 Q_0}{\lambda^* p}\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right) \quad (2.51c)$$

$$\bar{Z}'(\eta,p)=-T_0\lambda^*\left(1+\frac{1}{2a_0 p}\right)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right)-a_0 Q_0\left(1+\frac{1}{a_0 p}\right)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right) \quad (2.52c)$$

$$\bar{U}(\eta,p)=-\frac{T_0}{2a_0\lambda^*}\left[\frac{2a_0\tau_1}{p}-\frac{2a_0-\tau_1}{p^2}\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right)$$

$$-\frac{Q_0\tau_0\tau_1}{\left(\lambda^*\right)^2}\left[\frac{1}{p^2}+\frac{1}{p^3}\left(\frac{1}{\tau_1}+\frac{1}{\tau_0}-\frac{1}{a_0}\right)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right) \quad (2.53c)$$

$$\bar{U}'(\eta,p)=T_0\left(\tau_1+\frac{1}{p}\right)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right)$$

$$+\frac{Q_0\tau_0\tau_1}{\lambda^*}\left[1+\frac{1}{p}\left(\frac{1}{\tau_1}+\frac{1}{\tau_0}-\frac{1}{2a_0}\right)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right) \quad (2.54c)$$

and for normal load

$$\bar{Z}(\eta,p)=\frac{\varepsilon\sigma_0^*}{\left(\lambda^*\right)^2}\left[\frac{1}{p^2}-\frac{1}{a_0 p^3}\right]\left[1-\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right)\right]$$

$$+\frac{\tau_0 Q_0}{\lambda^* p}\left[1+\frac{1}{\tau_0 p}-\frac{1}{2a_0 p}\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right) \quad (2.55c)$$

$$\bar{Z}'(\eta,p)=-\frac{\varepsilon\sigma_0^*}{\lambda^* p}\left(1-\frac{1}{2a_0 p}\right)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right)-Q_0\left(\tau_0+\frac{1}{p}\right)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right) \quad (2.56c)$$

$$\bar{U}(\eta,p)=-\frac{\sigma_0^*}{\left(\lambda^*\right)^3}\left[\lambda^*\eta\tau_0\left(\frac{1}{p}+\frac{a_0-\tau_0}{a_0\tau_0 p^2}\right)-\varepsilon\tau_1\left(\frac{1}{p^2}+\frac{2a_0-3\tau_1}{2a_0\tau_1 p^3}\right)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right)\right]$$

$$-\frac{Q_0\tau_0\tau_1}{\left(\lambda^*\right)^2}\left[\frac{1}{p}+\frac{1}{p^2}\left(\frac{1}{\tau_1}+\frac{1}{\tau_0}-\frac{1}{a_0}\right)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right) \quad (2.57c)$$

$$\bar{U}'(\eta,p)=\frac{\varepsilon\sigma_0^*\tau_1}{\left(\lambda^*\right)^2}\left[\frac{1}{p}+\frac{1}{p^2}\left(\frac{1}{\tau_1}-\frac{1}{a_0}\right)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right)+\frac{\tau_0\sigma_0^*}{\left(\lambda^*\right)^2}\left[\frac{1}{p}+\frac{1}{p^2}\left(\frac{1}{\tau_0}-\frac{1}{a_0}\right)\right]$$

$$+\frac{Q_0\tau_0\tau_1}{\lambda^*}\left[1+\frac{1}{p}\left(\frac{1}{\tau_1}+\frac{1}{\tau_0}-\frac{1}{2a_0}\right)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\exp\left(-\lambda^* p\eta\right) \quad (2.58c)$$

Taking inverse Laplace transform of equations (2.51c) to (2.58c), we get for thermal shock

$$Z(\eta,\tau)=T_0 H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)+\frac{a_0 Q_0}{\lambda^*}H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right) \quad (2.75)$$

$$Z'(\eta,\tau)=-T_0\lambda^*\left[\delta(\tau-\lambda^*\eta)+\frac{1}{2a_0}H(\tau-\lambda^*\eta)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)$$

$$-Q_0\left[a_0\delta(\tau-\lambda^*\eta)+H(\tau-\lambda^*\eta)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right) \quad (2.76)$$

$$U(\eta,\tau)=-\left[\frac{T_0}{2a_0\lambda^*}\left\{2a_0\tau_1+(2a_0-\tau_1)(\tau-\lambda^*\eta)\right\}H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)\right]$$

$$-\frac{\tau_0\tau_1 Q_0}{\left(\lambda^*\right)^2}\left[(\tau-\lambda^*\eta)-\frac{(\tau-\lambda^*\eta)^2}{2}\left(\frac{1}{\tau_1}+\frac{1}{\tau_0}-\frac{1}{a_0}\right)\right]H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right) \quad (2.77)$$

$$U'(\eta,\tau)=T_0\left[\tau_1\delta(\tau-\lambda^*\eta)+H(\tau-\lambda^*\eta)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)$$

$$+\frac{\tau_0\tau_1 Q_0}{\lambda^*}\left[\delta(\tau-\lambda^*\eta)+\left(\frac{1}{\tau_1}+\frac{1}{\tau_0}-\frac{1}{a_0}\right)H(\tau-\lambda^*\eta)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right) \quad (2.78)$$

and for normal load

$$Z(\eta,\tau)=-\frac{\varepsilon\sigma_0^*}{\left(\lambda^*\right)^2}\left[\left\{(\tau-\lambda^*\eta)-\frac{(\tau-\lambda^*\eta)^2}{2a_0}\right\}H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)-\tau+\frac{\tau^2}{2a_0}\right]$$

$$+\frac{\tau_0 Q_0}{\lambda^*}\left[1-(\tau-\lambda^*\eta)\left(\frac{1}{\tau_0}-\frac{1}{2a_0}\right)\right]H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right) \quad (2.79)$$

$$Z'(\eta,\tau)=-\frac{\varepsilon\sigma_0^*}{\lambda^*}\left\{1-\frac{(\tau-\lambda^*\eta)}{2a_0}\right\}H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)$$

$$-Q_0\left[H(\tau-\lambda^*\eta)+\delta(\tau-\lambda^*\eta)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right) \quad (2.80)$$

$$U(\eta,\tau)=\frac{\sigma_0^*}{\left(\lambda^*\right)^3}\left[\lambda^*\eta\left\{\tau_0+\frac{a_0-\tau_0}{a_0}\tau\right\}-\varepsilon\tau_1\left\{(\tau-\lambda^*\eta)+\frac{(\tau-\lambda^*\eta)^2}{4a_0\tau_1}(2a_0-3\tau_1)\right\}\right]$$

$$\times H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)$$

$$-\frac{Q_0\tau_0\tau_1}{\left(\lambda^*\right)^2}\left[1+\left(\frac{1}{\tau_1}+\frac{1}{\tau_0}-\frac{1}{a_0}\right)(\tau-\lambda^*\eta)\right]$$

$$\times \quad H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right) \quad (2.81)$$

$$U'(\eta,\tau)=\frac{\sigma_0^*}{\left(\lambda^*\right)^2}\left[\left\{\tau_0+\frac{a_0-\tau_0}{a_0}\tau\right\}+\varepsilon\tau_1\left\{1+\frac{(\tau-\lambda^*\eta)}{a_0\tau_1}(a_0-\tau_1)\right\}\right]$$

$$H(\tau-\lambda^*\eta)\exp\left(-\frac{\lambda^*\eta}{2a_0}\right)$$

$$+\frac{Q_0\tau_0\tau_1}{\lambda^*}\left[\delta(\tau-\lambda^*\eta)+\left(\frac{1}{\tau_1}+\frac{1}{\tau_0}-\frac{1}{2a_0}\right)H(\tau-\lambda^*\eta)\right]\exp\left(-\frac{\lambda^*\eta}{2a_0}\right) \quad (2.82)$$

where

$$a_0=\frac{\tau_0+\varepsilon\tau_1}{1+\varepsilon} \quad \text{and} \quad \lambda^*=\sqrt{a_0(1+\varepsilon)}$$

Now, if we write $Q_0=0$, all the displacements and the temperature for both thermal shock and normal load are same as in (Sharma 1997).

Again, it is found that there are discontinuities at $\tau=\lambda_0\eta$ in the case of L-S theory, except the displacement for thermal shock. It is also found that at $\tau=\lambda^*\eta$, there are discontinuities in the case of G-L theory.

The jumps are the followings:

In the case of L-S theory, for thermal shock, we get from equation (2.67)

$$\left[Z\right]_{\tau=\lambda_0\eta}=\left(T_0+\frac{Q_0\tau_0}{\lambda_0}\right)\exp\left(-\frac{\tau}{2\tau_0}\right)$$

from equation (2.68)

$$\left[Z'\right]_{\tau=\lambda_0\eta}=-\left(\frac{T_0\lambda_0}{2\tau_0}+Q_0\right)\exp\left(-\frac{\tau}{2\tau_0}\right)$$

from equation (2.70)

$$\left[U'\right]_{\tau=\lambda_0\eta}=-\left(T_0+\frac{Q_0\tau_0}{\lambda_0}\right)\exp\left(-\frac{\tau}{2\tau_0}\right)$$

and for normal load. We get from equation (2.71)

$$\left[Z\right]_{\tau=\lambda_0\eta}=-\frac{\varepsilon\sigma_0^*}{1+\varepsilon}+\left(\frac{\varepsilon\sigma_0^*}{1+\varepsilon}-\frac{Q_0\tau_0}{\lambda_0}\right)\exp\left(-\frac{\tau}{2\tau_0}\right)$$

from equation (2.72)

$$\left[Z'\right]_{\tau=\lambda_0\eta}=-\left(\frac{\varepsilon\sigma_0^*}{2\lambda_0}+Q_0\right)\exp\left(-\frac{\tau}{2\tau_0}\right)$$

from equation (2.73)

$$\left[U\right]_{\tau=\lambda_0\eta}=\frac{\sigma_0^*\tau_0}{(1+\varepsilon)\lambda_0}\exp\left(-\frac{\tau}{2\tau_0}\right)$$

from equation (2.74)

$$\left[U'\right]_{\tau=\lambda_0\eta}=\frac{\sigma_0^*}{(1+\varepsilon)}+\left(\frac{\sigma_0^*}{(1+\varepsilon)}+\frac{Q_0\tau_0}{\lambda_0}\right)\exp\left(-\frac{\tau}{2\tau_0}\right)$$

In the case of G-L theory, for thermal shock, we get from equation (2.75)

$$\left[Z\right]_{\tau=\lambda^*\eta}=-\left(T_0+\frac{Q_0 a_0}{\lambda^*}\right)\exp\left(-\frac{\tau}{2a_0}\right)$$

from equation (2.76)

$$\left[Z'\right]_{\tau=\lambda^*\eta}=-\left(\frac{T_0\lambda^*}{2a_0}+Q_0\right)\exp\left(-\frac{\tau}{2a_0}\right)$$

from equation (2.77)

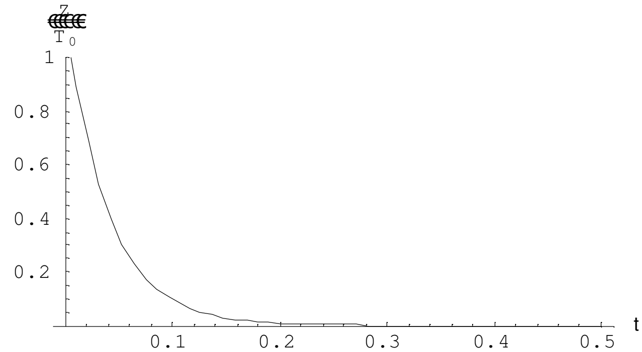

Figure 1. Temperature vs Time for thermal shock (L.S Theory)

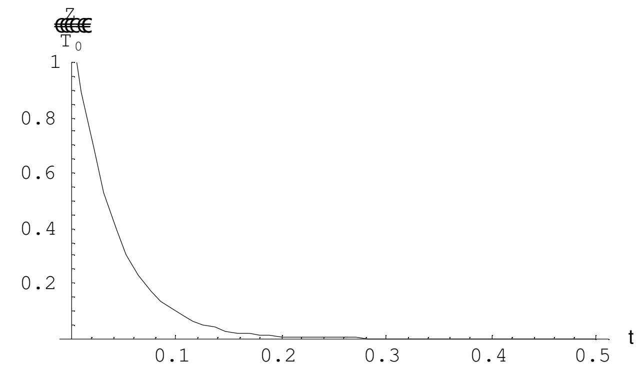

Figure 1a. Temperature vs Time for thermal shock (L.S Theory),considering the source term $Q_0 = 0$

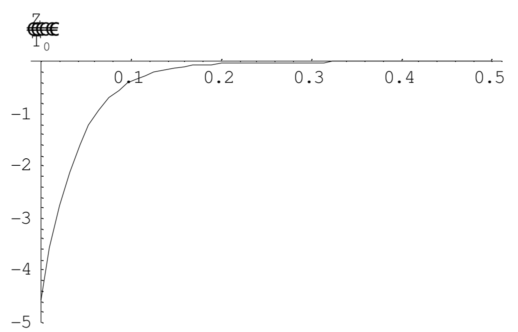

Figure 2. Temperature Gradient vs Time for thermal shock (L.S Theory)

$$[U]_{\tau=\lambda^*\eta} = -\frac{T_0\tau_1}{\lambda^*}\exp\left(-\frac{\tau}{2a_0}\right)$$

from equation (2.78)

$$[U']_{\tau=\lambda^*\eta} = \left(T_0 + \frac{Q_0\tau_0\tau_1}{\lambda^*}\left(\frac{1}{\tau_1} + \frac{1}{\tau_0} - \frac{1}{2a_0}\right)\right)\exp\left(-\frac{\tau}{2a_0}\right)$$

and for normal load, we get from equation (2.79)

$$[Z]_{\tau=\lambda^*\eta} = \left[\frac{\varepsilon\sigma_0^*}{(\lambda^*)^2}\left(\frac{\tau^2 - 2a_0\tau}{2a_0}\right) + \frac{Q_0\tau_0}{\lambda^*}\right]\exp\left(-\frac{\tau}{2a_0}\right)$$

from equation (2.80)

$$[Z']_{\tau=\lambda^*\eta} = -\left[\frac{\varepsilon\sigma_0^*}{\lambda^*} + Q_0\right]\exp\left(-\frac{\tau}{2a_0}\right)$$

from equation (2.81)

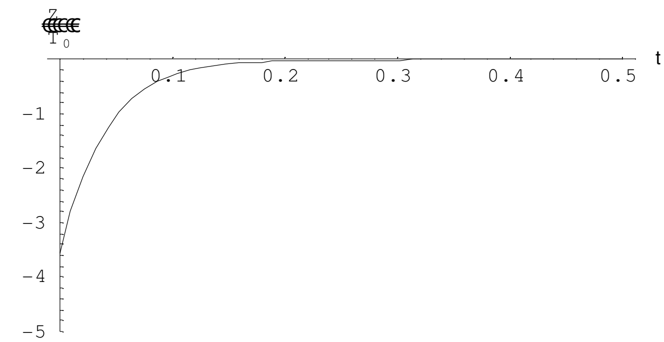

Figure 2a. Temperature Gradient vs Time for thermal shock (L.S Theory) considering the source term $Q_0 = 0$

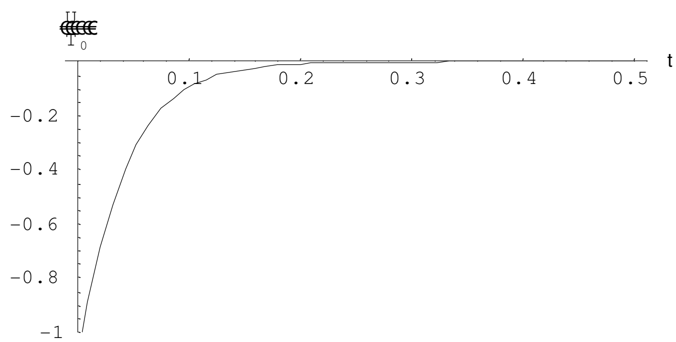

Figure 3. Displacement Gradient vs Time for thermal shock (L.S Theory)

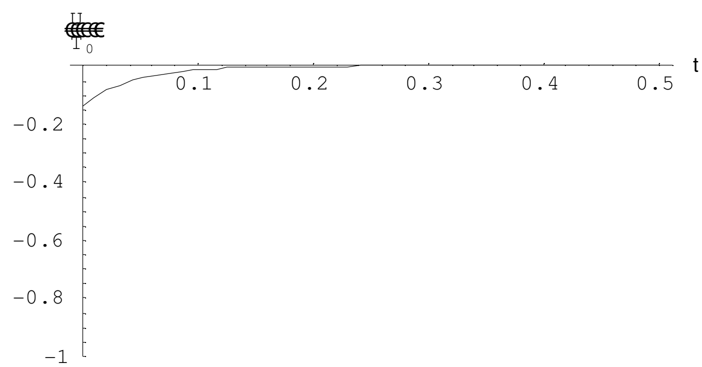

Figure 3a. Displacement Gradient vs Time for thermal shock (L.S Theory) considering the source term $Q_0 = 0$

$$[U]_{\tau=\lambda^*\eta} = -\left[\frac{\sigma_0^*\tau}{(\lambda^*)^3}\left(\tau_0 + \frac{a_0-\tau_0}{a_0}\tau\right) - \frac{Q_0\tau_0\tau_1}{(\lambda^*)^2}\right]\exp\left(-\frac{\tau}{2a_0}\right)$$

from equation (2.82)

$$[U]_{\tau=\lambda^*\eta} = -\left[\frac{\sigma_0^*}{(\lambda^*)^2}\left(\tau_0 + \frac{a_0-\tau_0}{a_0}\tau\right) + \left\{\frac{\varepsilon\tau_1\sigma_0^*}{(\lambda^*)^2} + \frac{Q_0\tau_0\tau_1}{\lambda^*}\left(\frac{1}{\tau_1} + \frac{1}{\tau_0} - \frac{1}{2a_0}\right)\right\}\right]\exp\left(-\frac{\tau}{2a_0}\right)$$

NUMERICAL RESULTS AND DISCUSSIONS

Here we prefer to determine the state in the state-space domain numerically, for a fixed value of a space variable and for varying time.

Taking values of the constants τ_0, ε, λ_0 as

τ_0 = 0.02 sec, ε = 0.0168, λ_0 = 0.1426,

we can find

$a_0 = \dfrac{\tau_0 + \varepsilon\tau_1}{1+\varepsilon}$ and $\lambda^* = \sqrt{a_0(1+\varepsilon)}$

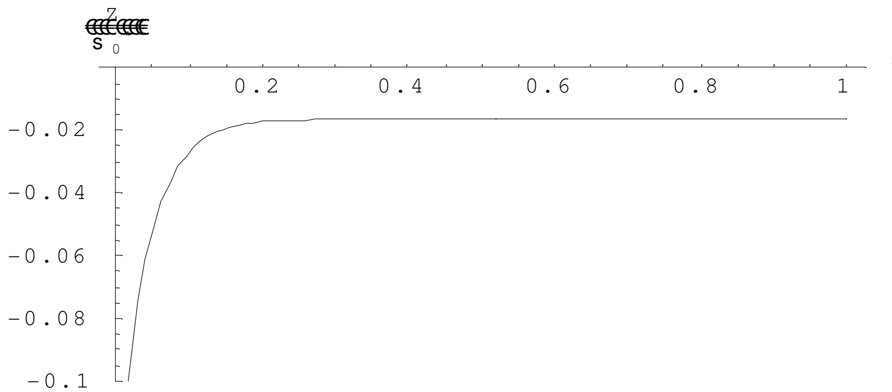

Figure 4. Temperature vs Time for normal load (L.S Theory)

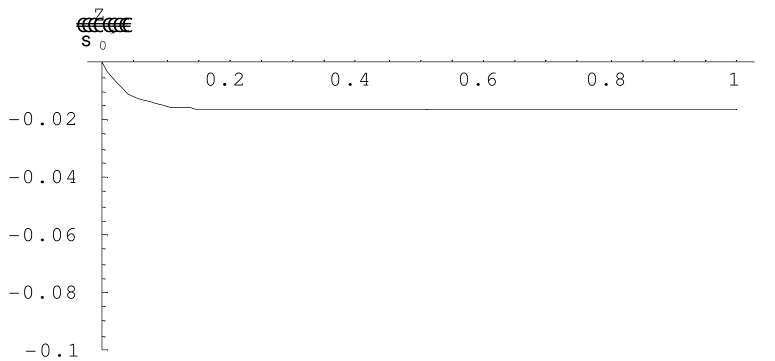

Figure 4a. Temperature vs Time for normal load (L.S Theory) considering the source term $Q_0 = 0$

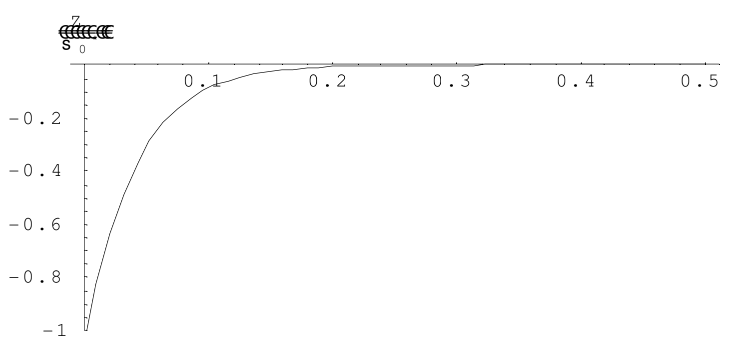

Figure 5. Temperature Gradient vs Time for normal load (L.S Theory)

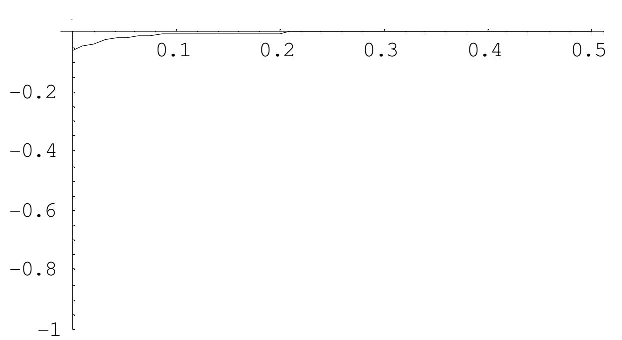

Figure 5a. Temperature Gradient vs Time for normal load (L.S Theory) considering the source term $Q_0 = 0$

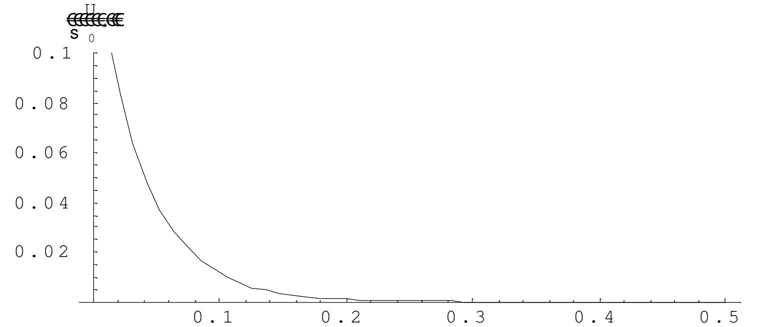

Figure 6. Displacement vs Time for normal load (L.S Theory)

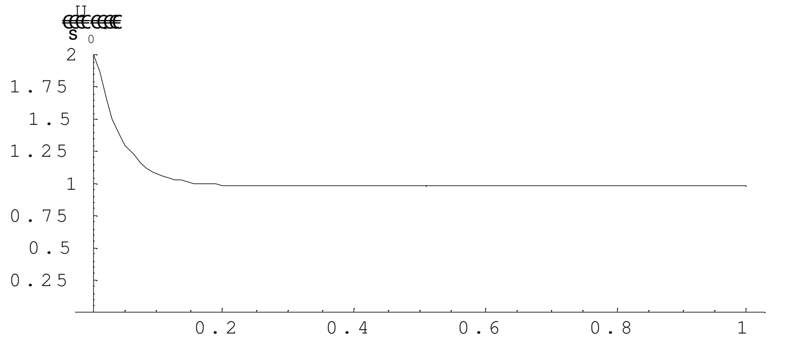

Figure 7. Displacement Gradient vs Time for normal load (L.S Theory)

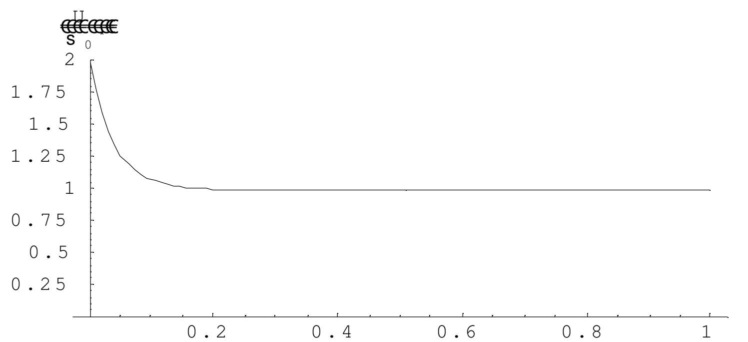

Figure 7a. Displacement Gradient vs Time for normal load (L.S Theory) considering the source term $Q_0 = 0$

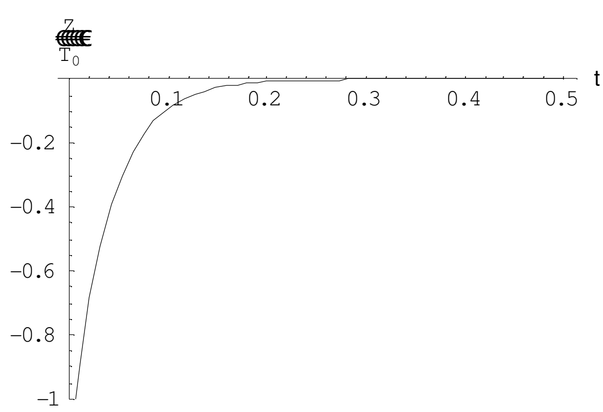

Figure 8. Temperature vs Time for thermal shock (G.L Theory)

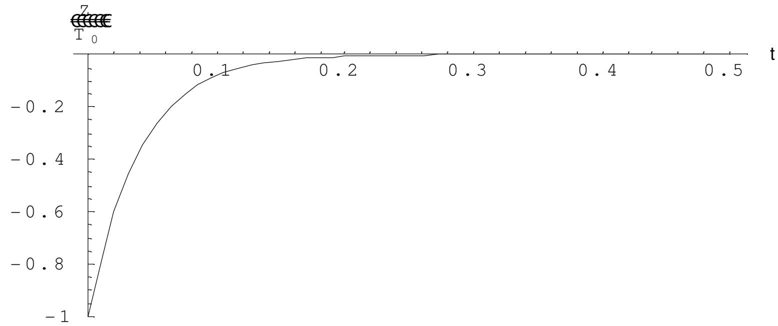

Figure 8a. Temperature vs Time for thermal shock (G.L Theory) considering the source term $Q_0 = 0$

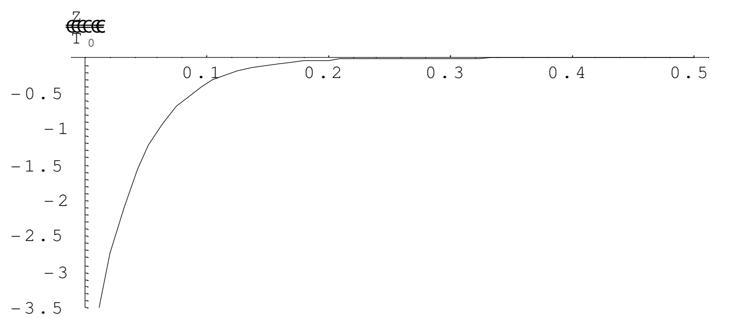

Figure 9. Temperature Gradient vs Time for thermal shock (G.L Theory)

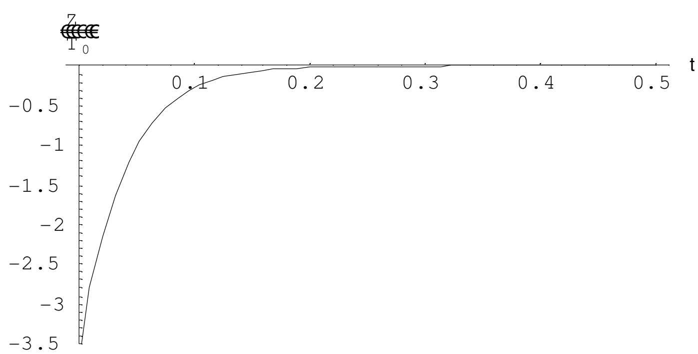

Figure 9a. Temperature Gradient vs Time for thermal shock (G.L Theory) considering the source term $Q_0 = 0$

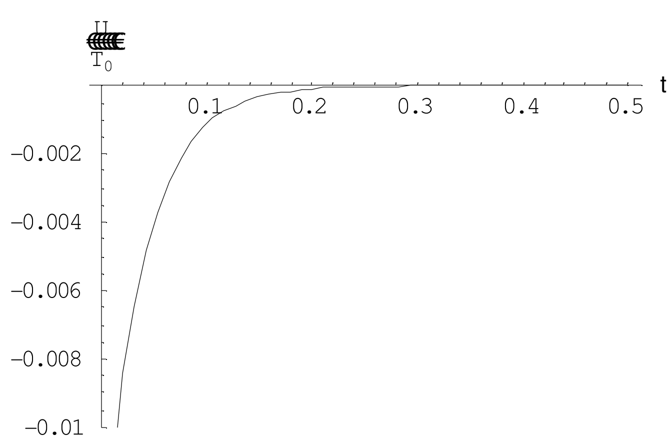

Figure 10. Displacement vs Time for thermal shock (G.L Theory)

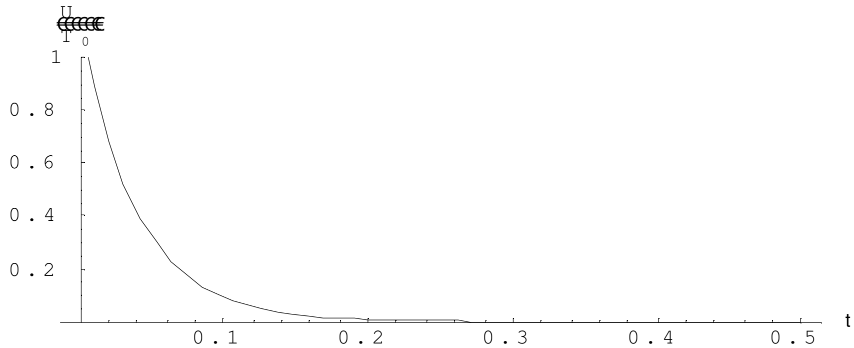

Figure 11. Displacement Gradient vs Time for thermal shock (G.L Theory)

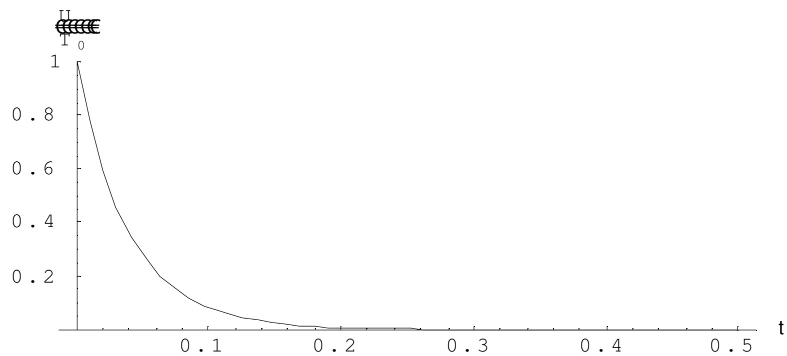

Figure 11a. Displacement Gradient vs Time for thermal shock (G.L Theory) considering the source term $Q_0 = 0$

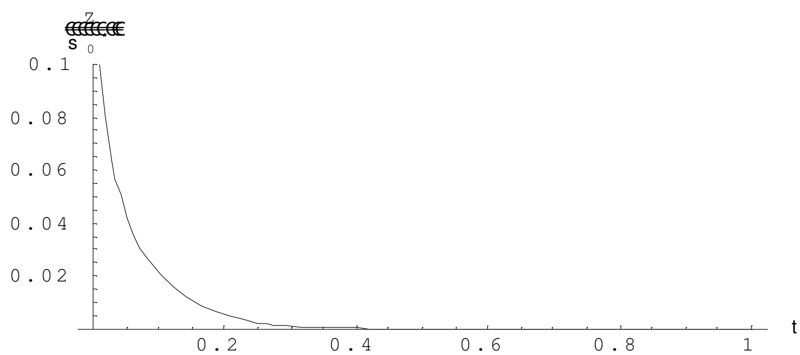

Figure 12. Temperature vs Time for normal load (G.L Theory)

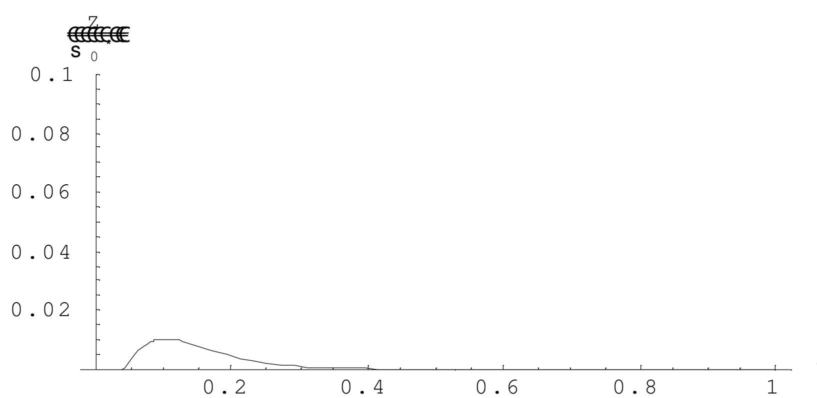

Figure 12a. Temperature vs Time for normal load (G.L Theory) considering the source term $Q_0 = 0$

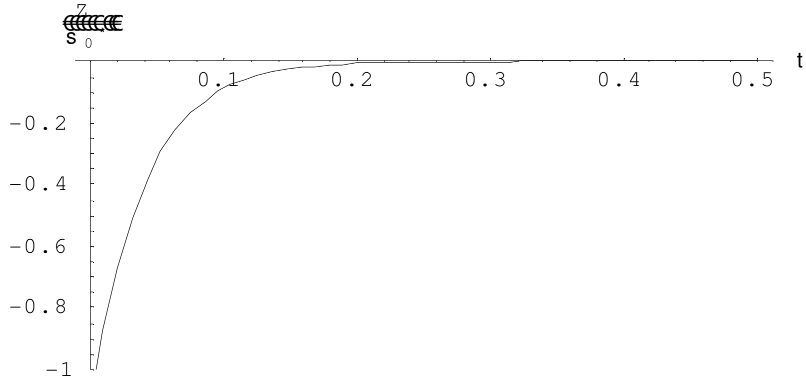

Figure 13. Temperature Gradient vs Time for normal load (G.L Theory)

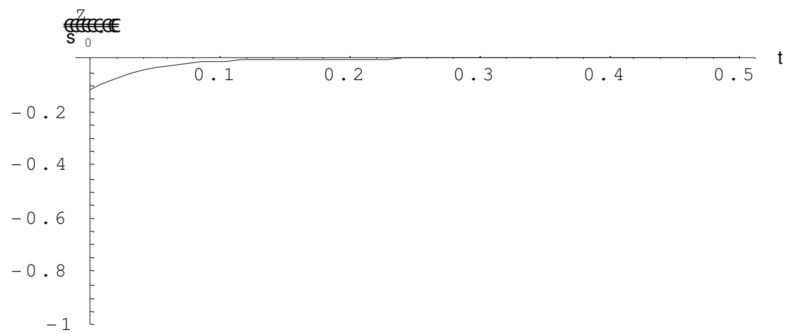

Figure 13a. Temperature Gradient vs Time for normal load (G.L Theory) considering the source term $Q_0 = 0$

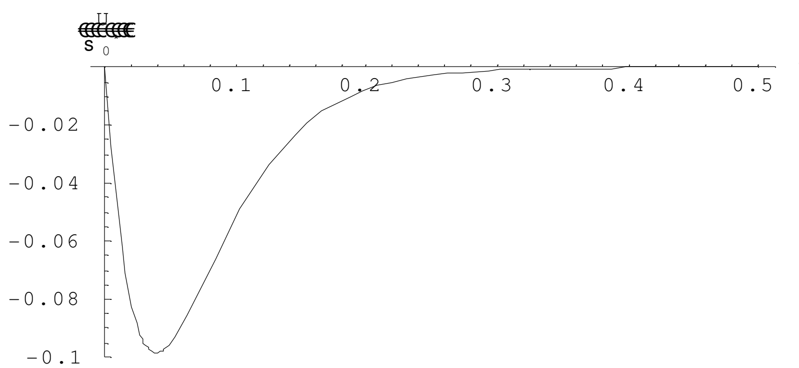

Figure 14. Displacement vs Time for normal load (G.L Theory)

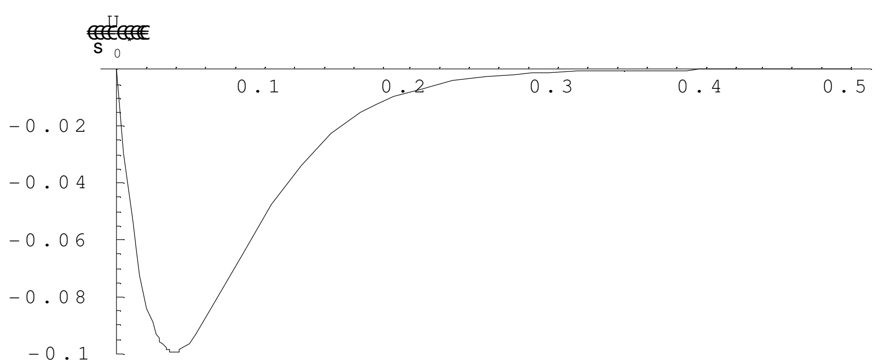

Figure 14a. Displacement vs Time for normal load (G.L Theory) considering the source term $Q_0 = 0$

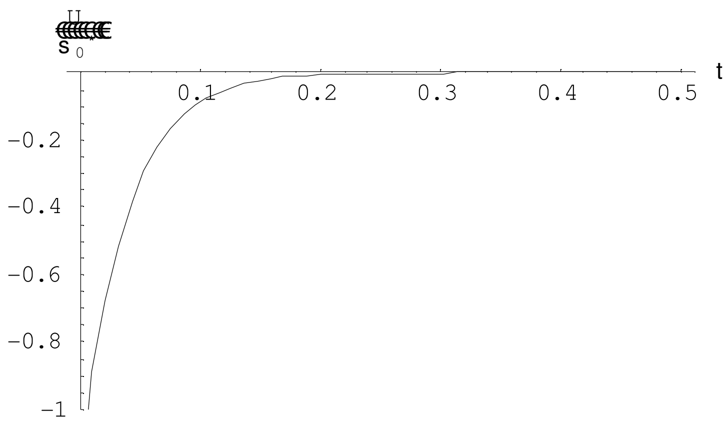

Figure 15. Displacement Gradient vs Time for normal load (G.L Theory)

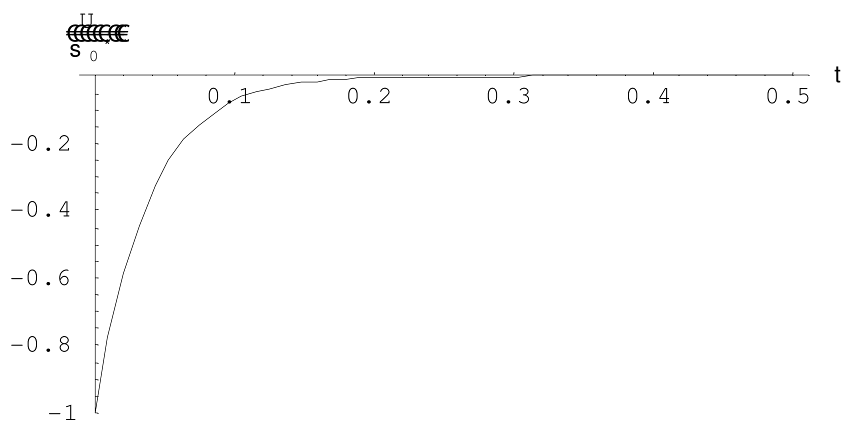

Figure 15a. Displacement Gradient vs Time for normal load (G.L Theory) considering the source term $Q_0 = 0$

The above values are used and the graphs are plotted.

In case of L-S theory, it is seen from the graphs that characteristics of the parameters under consideration are nonlinear in nature and are different from those of G-L theory. From Figure 1, we see that temperature decreases as time increases in case of thermal shock. Temperature gradient and displacement gradient increase from negative value as time increases and ultimately tends to zero in case of thermal shock as is found in Figure 2 and 3.

In case of normal load, temperature increases from negative value but remains negative and steady after some time, as is seen from Figure 4. From Figure 5, it is seen that if there be no source term, the result is same as is found in (Hetnarski, 1964), but nature of the graph remains the same.

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